British Society for
Research into
Learning Mathematics

Research Review of BSRLM

2003-2017

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List of Acronyms used in Review

BCME	British Congress of Mathematics Education
BSRLM	British Society for Research into Learning Mathematics
CERME	Congress of the European Society for Research in Mathematics Education
CMEP	Cambridge Mathematics Education Project
CPD	Continuing Professional Development
DCSF	Department for Children, Schools and Families (UK: 2007 – 2010)
DfE	Department for Education (UK: 2010 – present)
DfES	Department for Education and Skills (UK: 2001 – 2007)
DMT	Dynamic Mathematical Technology
DNL	Double Number Line
EAL	English as an Additional Language
ECR	Early Career Researcher
EEF	Education Endowment Foundation
ERME	European Society for Research in Mathematics Education
ESM	Educational Studies in Mathematics (Journal)
EYFS	Early Years Foundation Stage (birth to 5-year-olds)
FE	Further Education (Post-16)
GCSE	General Certificate in Secondary Education (England & Wales: exams at age 16)
HE	Higher Education (tertiary education 18+)
HEI	Higher Education Institution
ICCAMS	Increasing Competence and Confidence in Algebra and Multiplicative Structures
ITT / ITE	Initial Teacher Training / Education
JRME	Journal for Research in Mathematics Education
KS1	Key Stage 1 (England & Wales, ages 5-7)
KS2	Key Stage 2 (England & Wales, ages 7-11)
KS3	Key Stage 3 (England & Wales, ages 11-14)
KS4	Key Stage 4 (England & Wales, ages 14-16)
KS5	Key Stage 5 (England & Wales, ages 16-18), also referred to as the Sixth Form
LEA	Local Education Authority
MaST	Mathematics Specialist Teacher programme
MESC	Mathematics Education Subject Classification list
MPTK	Mathematical Pedagogical Technology Knowledge
NC	National Curriculum (England & Wales)
NCETM	National Centre for Excellence in the Teaching of Mathematics
NNS	National Numeracy Strategy
Ofsted	Office for Standards in Education, Children's Services and Skills
PCK	Pedagogical Content Knowledge
PNS	Primary National Strategy
RECME	Researching Effective CPD in Mathematics Education
RME	Research in Mathematics Education (Journal of BSRLM)
SATs	Statutory Assessment Tests (England only: conducted at the end of KS1 and KS2)
SEN / SEND	Special Educational Needs / Special Educational Needs and Disabilities
TA	Teaching Assistant
TIMSS	Trends in International Mathematics and Science Study
WG	Working Group (ongoing groups meeting at BSRLM)

1 Executive Summary

1.1 Introduction

This review is a commissioned analysis of the 773 Proceedings arising from Day Conferences and which were published by the British Society for Research into Learning Mathematics (BSRLM) during the last 15 years (2003-2017). It builds on a previously commissioned review of the British Society for Research into Learning Mathematics' conference Proceedings from 1995-2002. The 773 Proceedings underpinning this review represent the outputs of approximately 1375 sessions (Research Papers, Research Workshops and Working Groups) presented at 45 conferences at locations across the UK and Ireland.

1.2 Methodology

This review was undertaken by a team of four researchers. It was conducted in two phases: a survey of the full corpus (covering 2003-2017) and an in-depth analysis and discussion of key themes arising from this survey. The full corpus survey involved the construction of a database and capturing of descriptive characteristics for each Proceeding (paper type, approach, phase, study population, data, research methods, and analysis). Further, each Proceeding was categorised using the Mathematics Education Subject Classification (MESC) list. Descriptive statistics were used to explore trends within and between characteristics. Further, splitting the review period into three time-phases (2003-2007, 2008-2012, 2013-2017) allowed the team to identify any changes over time. The frequency of keywords and thematic groupings of keywords informed the vital areas to be explored within thematic analysis. The reviewers' tracking of narratives that developed through connected reports of research within the Proceedings also played a role in the selection of key themes to discuss.

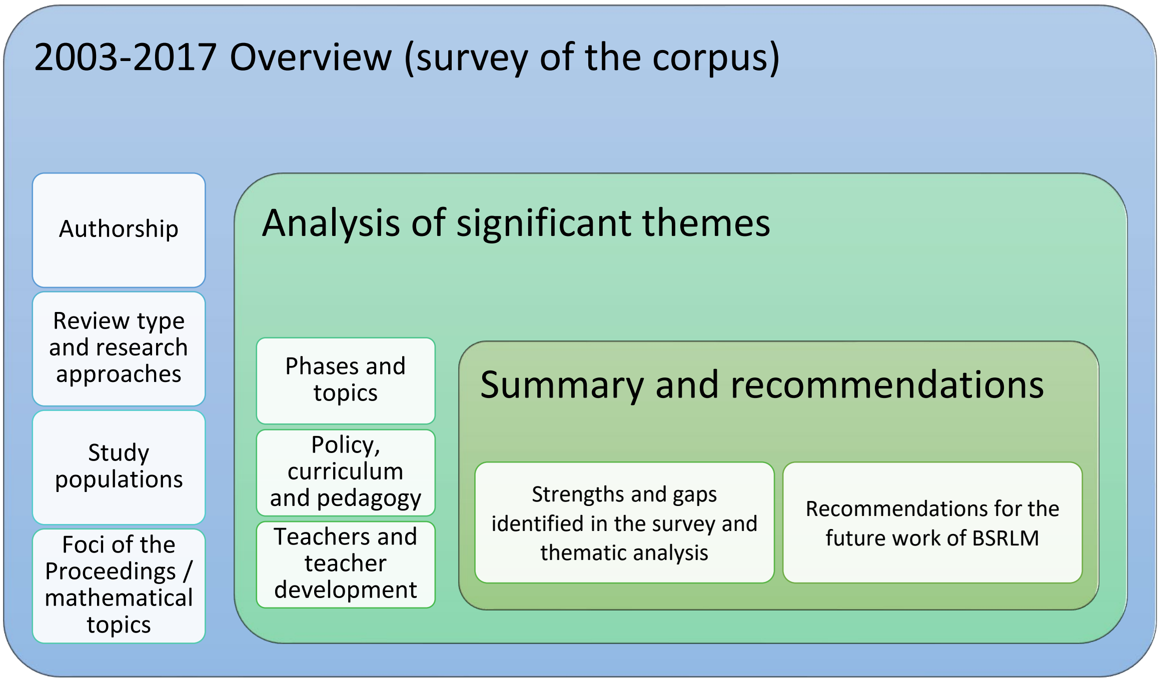
The outcomes of both phases of the review – in which strengths and gaps were noted – enabled the team to produce a series of seven recommendations for future work and developments that the British Society for Research into Learning Mathematics and its Executive might consider.

1.3 Review structure

The review structure reflects the methodological approach of the review. A full overview of the statistical analysis of the 773 Proceedings is provided, followed by an in-depth discussion of three broad themes:

- Phases and Topics in Mathematics Education
- Policy, Curriculum and Pedagogy
- Teachers and Teacher Development

Broadly, the shape of the review can be categorised thus:



The review concludes with seven specific recommendations in relation to the future work of the British Society for Research into Learning Mathematics. These recommendations are offered in the spirit of responding to identified gaps, ensuring the British Society for Research into Learning Mathematics responds to all learners, supports a changing membership, including the growing number of teacher-researchers, and invests in the development of the next generation of mathematics education researchers. In doing so, the British Society for Research into Learning Mathematics has the potential to continue to go from strength to strength, providing an important platform for the dissemination of significant research in mathematics education.

1.4 Key findings and recommendations

Building on the previous review (1995-2002), we note that many of the concerns raised in that document have been addressed, fully or partly. We see evidence of a growing membership, not only in terms of actual numbers, but also in terms of reach, with more teacher-practitioners, international researchers, and novel partnerships emerging. There is evidence of an increased focus on the primary phase of education and also on studies examining the involvement of teachers in their professional development. Policy changes also appear to have had a significant impact on the foci of research conducted.

The themes which emerged in the present review strongly align with those coming through in international reviews, suggesting the work conducted within BSRLM to be representative of, and hence able to speak to and with, the wider field of mathematics education.

While the corpus was found to have significant strengths, addressing many of the weaknesses identified in the previous review, we draw attention to some areas of concern in relation to absences or weaknesses in coverage, particularly the limited number of the Proceedings focussing on the Early Years Foundation Stage or learners with Special Educational Needs and Disabilities.

The seven specific recommendations of this review in relation to the focus and dissemination of the British Society for Research into Learning Mathematics research are summarised as:

i. A greater emphasis on identified population gaps (Early Years Foundation Stage, learners with Special Educational Needs and Disabilities, Further and Adult Education), promoted through Working Groups or Day Conferences with a ‘special focus’.

ii. Exploration of the potential of developing links with Early Years professional networks to address this population gap.

iii. Discussion by the Society of ways it may support methodological innovation including the scaling up of small-scale research.

iv. Discussion by the Executive of its role in supporting practitioner-researchers and in disseminating research to a teacher audience.

v. Establishing ways in which the longevity, membership, and impact of Working Groups can be enhanced.

vi. The development of keyword searching of the Proceedings on the website to support wider readership/citation of the Proceedings.

vii. The inclusion of further sessions at Day Conferences supporting writing and dissemination.

2 Introduction and Review Structure

This review was commissioned by the British Society for Research into Learning Mathematics in order to give a critical reflection on BSRLM research from 2003 to 2017. It offers a survey of all Proceedings published over the last 15 years[1], in addition to an in-depth discussion of key themes arising from this survey. This review also highlights strengths and identifies gaps in terms of coverage (including content, focus and methodological approach) across the total profile of studies. We suggest ideas for further study and point to possible developments for the future of the Society.

BSRLM organises three Day Conferences each year. In Spring 2005 and Spring 2010, BCME6 and BCME7 replaced the Day Conferences; papers from these conferences are included here with analysis suggesting no significant differences in the authorship, approach, or focus of BCME papers compared with traditional BSRLM Proceedings. The 45 conferences within the bounds of this review allowed for the dissemination of approximately 1375 sessions. Around 56% of conference sessions were subsequently published as Proceedings, resulting in the 773 individual Proceedings which make up this review. These Proceedings demonstrate the inclusive nature and diversity of BSRLM; while the vast majority resemble the 'traditional' research paper, we also see theoretical discussions (e.g. Watson, 2010), methodological debates (e.g. Coles, 2007) and systematic literature reviews (e.g. Kyriacou & Goulding, 2004) to name but a few. Although this review only focuses on the Proceedings and not the full range of BSRLM presentations, an analysis of the conference abstracts versus the published Proceedings for the last five years reveals no obvious skew in the focus or authorship of the Proceedings which were published (although see our Working Group analysis, Section 4.2.1 which suggests that WG presentations / discussion groups are less likely to be translated into a published form).

The structure of this review reflects the distinct stages of our work. Following the exposition of our review methodology (Section 3) we present an overview of the full corpus including: authorship, study population, paper types, research approaches and topic focus (Section 4). Engaging in this overview led to us identifying themes warranting in-depth attention. Subsequently, Sections 5, 6 and 7 cover:

- Phases and Topics in Mathematics Education
- Policy, Curriculum and Pedagogy
- Teachers and Teacher Development

We conclude in Section 8 with a discussion of the implications of the findings of the review and a series of recommendations for the future work of BSRLM, its Executive and its membership.

2.1 Relationship with the international field

As Inglis & Foster (2018, p.462) note, mathematics education research "has a long history" with the authors identifying 1968 as the crucial year in which a 'new phase' emerged and international journals in the field began publishing. BSRLM could therefore be considered relatively young, only

[1] All of the Proceedings from 1993 to present are available open-access at: http://www.bsrlm.org.uk/publications/proceedings-of-day-conference/

being established in 1985. However, in its 30+ years, it has developed into an internationally recognised forum for the dissemination of mathematics education research. This international element was strengthened in 2008 – during the period of this present review – by the inception in 2008 of *Research in Mathematics Education (RME)*, an international, refereed, English-language journal and the official journal of BSRLM (Rowland & Nardi, 2008; Nardi & Rowland, 2008). Publishing research from contributors internationally feasibly enhanced the status of BSRLM and its Proceedings, a number of which have been incorporated into RME through its 'Current Reports' section and full papers.

A previous review (Nickson, 2003) examined the BSRLM proceedings from 1995-2002. Nickson's key findings were that the Proceedings predominantly reported empirical classroom-based research, examining what pupils and students were doing in the classroom.[2] Little attention appeared to be paid to research-informed CPD. Of the classroom-based studies, the majority examined secondary classrooms, with Nickson recommending a stronger primary focus in future work.[3] Perhaps unsurprising given the secondary classroom focus, many studies focused on mathematical topic areas, with a preponderance of the Proceedings looking at algebra and geometry (and to a lesser extent, numeracy, potentially influenced by the inception of the National Numeracy Strategy (NNS) in England).[4] Interestingly, although algebra was a strong focus, Nickson's nuanced analysis noted a paucity of studies examining either pre-algebra or the teaching of algebra with technology. We engage with Nickson's findings in the present review, highlighting changes and continuities across the two time periods.

During the time-period covered by the present review, there has been an increasing interest in developing a deeper understanding of the 'state of the art' of mathematics education research, particularly in an international context. It could be argued that this was instigated by the publication, in 2002, of '*The story of ESM'* (Hanna, & Sidoli, 2002, see also: Lerman, Xu & Tsatsaroni, 2003), profiling the content and focus of papers and examining 35 years of developments in *Educational Studies in Mathematics* and – of course – the field of mathematics education. Since BSRLM commissioned our review in 2017, two further significant reviews of the state of the art of mathematics education research internationally (with a focus on articles again published in *Educational Studies in Mathematics* as well as in the *Journal for Research in Mathematics Education)* (Inglis & Foster, 2018) and in Europe through the *European Society for Research in Mathematics Education (ERME)* (Dreyfus et al., 2018) have been published. While it is not possible to do justice here to the rich analysis presented in these three reviews, it is worth noting that all three present the field as dynamic, realigning to reflect changing concerns, new developments and wider changes in the educational research climate. Where appropriate, we point to congruencies between our review – with its inherent UK focus – and the international context presented in these wider reviews.

[2] Throughout this review we use 'pupil' to indicate learners aged from 3 to 16 and 'student' to indicate learners in post-16 settings.

[3] In England and Wales, Primary education covers ages 5-11 (Years 1-6) and Secondary education covers ages 11-16/18 (Years 7-11/13).

[4] The National Numeracy Strategy arose from a concern over poor numeracy standards in England. Led by the Numeracy Task Force, the strategy outlined both content and pedagogic approaches – prescribing a 'Numeracy Hour' each day – for all pupils from Reception (ages 4-5) to Year 6.

3 Methodology

Initial analysis of the 773 Proceedings produced between 2003 and 2017 was completed using a systematic approach. 19 specific characteristics were identified for each Proceeding and added to a database. These characteristics included basic descriptive characteristics such as Proceeding title, author(s), author location(s), keywords, study country, and conference location. Keywords were assigned by coders when none were present (e.g. all of the Proceedings from 2003 up to and including March 2008) or when the keywords assigned were not suitable or useful (e.g. instances when "mathematics education" was listed as a keyword). Characteristics describing the nature of the Proceeding were also identified such as paper type, approach, phase, study population, data, research methods, and analysis (see Table 1 for examples). The research team, comprising four mathematics education lecturers, also included two codes for each Proceeding which related directly to the Mathematics Education Subject Classification (MESC) list. The MESC scheme includes 16 broad codes which the research team used to classify the overall focus of each Proceeding. For example, these broad codes included *Psychology of Mathematics Education* (e.g. cognitive processes, language and communication, affective aspects) and *Education and Instruction in Mathematics* (e.g. goals of mathematics teaching, assessment, teaching methods and classroom techniques). Using this MESC list complemented the tracking of keywords, thus offering a further means of identifying the focus of each Proceeding.

Characteristics	Examples
Paper Type	empirical, literature review, theoretical, Working Group report
Approach	case-study, ethnography, grounded research, meta-analysis, thought piece
Phase	EYFS, Primary KS1, Secondary KS4, HE, Adult education
Study Population	pupils/students doing mathematics, trainee teachers, workplace, practicing teachers
Data	quantitative, qualitative, mixed methods
Research Methods	interview, video, observation, work scrutiny
Analysis	discourse analysis, thematic analysis, descriptive statistics, inferential statistics

Table 1: Selection of characteristics coded and examples of each

The research team commenced coding each of the 2002 Proceedings as a pilot study using the characteristics outlined. The team met regularly to compare notes on key elements of the coding process and agree any adjustments to the processes or codes involved. This process ensured that the general approach taken to this phase of analysis was refined and improved before coding of the corpus produced during the period of 2003-2017. A range of the Proceedings were also selected and coded independently by each of the four coders in the research team in order to check for inter-coder reliability, i.e. checking that the research team consistently entered the same data in each field (Cohen, Manion & Morrison, 2011). The team met to compare codes upon completion of this independent coding and discussed any differences observed, thus informing future coding and refining terms where necessary. Thereafter, any difficulties in coding were flagged on the database and discussed in person or via e-mail with the team. The research team initially planned to code each Proceeding using the relevant abstracts but it was quickly determined that scrutiny of the full Proceedings would be necessary in order to complete coding accurately. Notes were also kept in relation to the suitability for a Proceeding to be included in the thematic analysis of the corpus.

In the latter stages of coding the corpus, it was agreed that thematic groupings of keywords should be added in order to transition from a list of 174 keywords to a more manageable list of 14 thematic groupings in preparation for analysis. It must be noted that these groupings were not mutually exclusive and, as a result, each Proceeding was typically assigned multiple themes. The analysis of the data was completed by compiling descriptive statistics based on the categories previously outlined in three five-year bands (2003-2007, 2008-2012, 2013-2017). These statistics were used to determine trends in the manner, focus, and origin of the Proceedings originating from research presented at BSRLM conferences during this time period. The frequency of keywords and thematic groupings of keywords also informed the vital areas to be explored within thematic analysis. The authors' tracking of narratives which developed through connected reports of research within the Proceedings also played a role in the selection of key themes to discuss. Similarly, the impact of the associated projects or research beyond the BSRLM conference setting and the importance of the area of research to developments at that time were crucial factors in selecting the Proceedings for inclusion in thematic analysis. The discussion of the ICCAMS and Underground Mathematics projects later in this review are prime examples of the development of such narratives in multiple conference Proceedings over a number of years (see Sections 5.3.3 and 5.4.1). Such projects also had meaningful impact outside the BSRLM conference setting.

4 2003-2017 Overview

As discussed in Section 3, the first stage of our analysis involved a statistical overview of the corpus. The preliminary results of this analysis were presented previously (Marks, Barclay, Barnes & Treacy, 2018). In this section we provide an overview of the key findings as a primer to the remainder of this review in which we develop the key themes, repeated ideas and areas of significance arising from our analysis and reading of the Proceedings.

4.1 Authorship and study location

Perhaps unsurprisingly given that BSRLM advocates for UK mathematics education, with Rowland & Nardi (2008, p.1) noting a focus of RME being to “retain the heritage of high quality, UK-based research”, 83% of the authors of the Proceedings were affiliated to UK institutions, with approximately 77% of studies, where stated, conducted in the UK.[5] However, the authors of the Proceedings represent 35 countries globally; 17% of the Proceedings are authored by or in collaboration with individuals from outside the UK, representing countries from all continents (except Antarctica!). This is an increasing trend, with international writers accounting for 14% of authors during 2003-2007, rising to 19% in both 2008-2012 and 2013-2017. Notably, a sizeable number of the Proceedings are written by authors from the Republic of Ireland[6] and Turkey. A potential weakness exists in cross-country collaboration, with just 5% of studies conducted across more than one country. Of the Proceedings from the UK, it is perhaps concerning that Scotland, Wales and Northern Ireland accounted for only 0.7%, 0.8% and 0.1% respectively of the UK Proceedings, with over 60% of the UK Proceedings originating from authors affiliated to institutions in the south of England (Figure 1). This may carry implications for the generalisability of findings (discussed further in Section 8).

Authors are predominantly affiliated to HEIs, with just 6% of the Proceedings having one or more authors who identified their primary affiliation as being outside of an HEI (e.g. a school, college, LEA, external agency, etc.). Of these, 45% were written in collaboration with an HEI partner, perhaps suggesting a need to support BSRLM members not affiliated to an HEI to present their work.

[5] Over 10% of the Proceedings did not state the country/ies where the study was conducted or to which the theoretical discussion related. While in some cases we could deduce the location from our knowledge of the authors and the field, this was not possible in a number of cases. We encourage future authors of Proceedings to explicitly state the study country in their writing.

[6] Although the Day Conference held in Dublin in 2015 accounted for approximately a third of these Proceedings, authors from the Republic of Ireland were represented across the Proceedings arising from other conference locations.

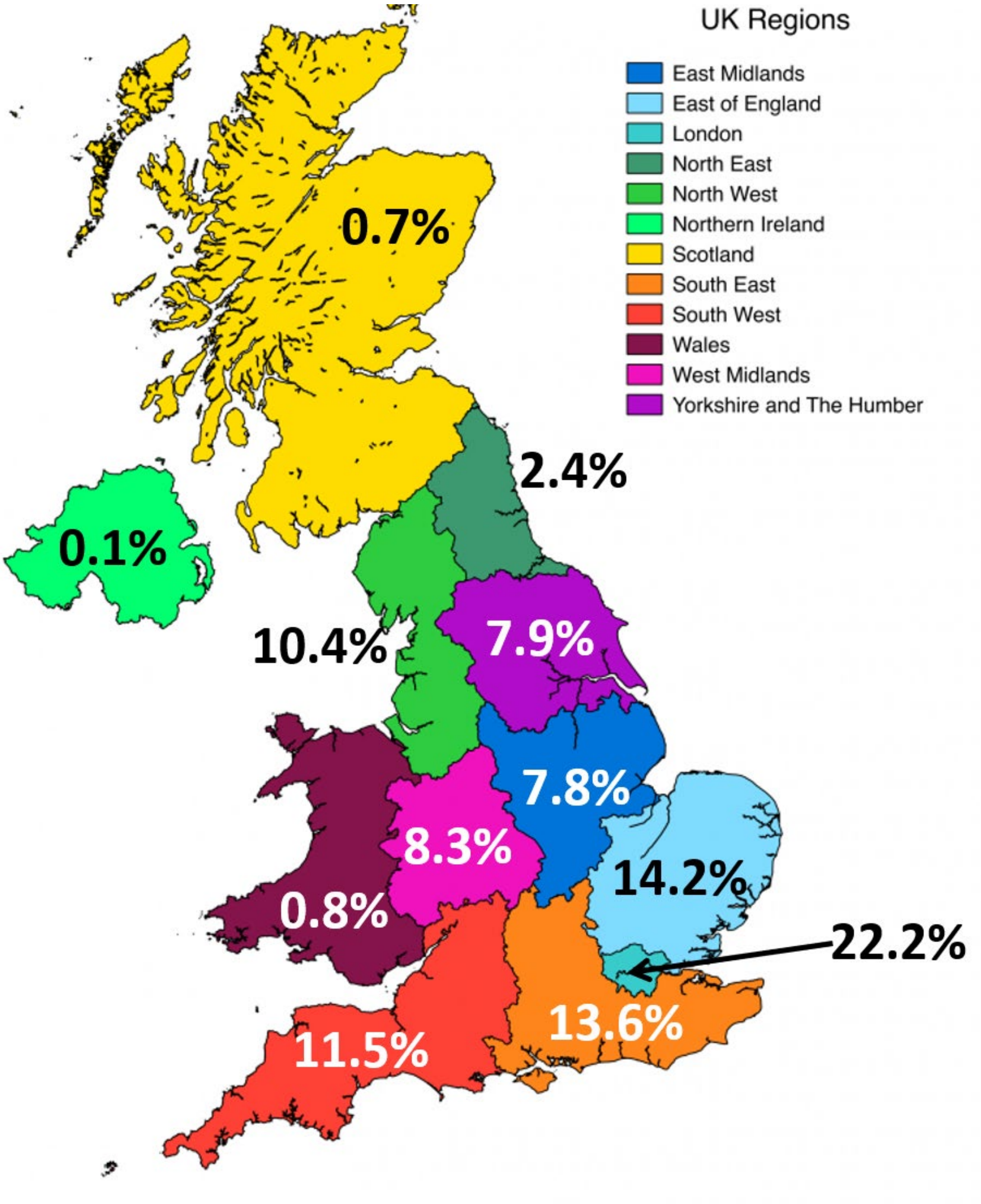


Figure 1: Authorship of the Proceedings by UK region

4.2 Categories and research methods

In Nickson's (2003) previous review, she noted that many studies involved classroom-based empirical research. Our review suggests a strong degree of continuity here, with 70% of the Proceedings from 2003-2017 presenting outcomes from empirical studies. Beyond these, 13% of the Proceedings were reports of a variety of types including from Working Groups (which accounted for 4% of the corpus), while 8% were theoretical discussions such as Rowlands & Graham's (2005) discussion using schema theory to put forth arguments around conceptual change in mechanics. Just 3% of the corpus were either literature reviews such as Kyriacou & Goulding's (2005) review of motivation in KS4 mathematics or policy reviews/reports such as Brown et al.'s (2006) report on their “*Developing Curriculum Pathways in Mathematics*” project assessing possible curriculum and assessment structures for post-14 mathematics.

The imbalance between empirical studies and other works might be thought to be an artefact of the nature of BSRLM and its Day Conferences. However, the split between empirical and other works appears to be in line with other large-scale reviews of published mathematics education papers internationally; Hanna & Sidoli’s (2002) review of ESM papers from 1990-1998 found 77% to be empirical, while Hart et al.’s (2009) study across six journals from 1995-2005 found 81% to be empirical.

4.2.1 BSRLM Working Groups

Working Groups (WGs) are described as groups which meet periodically – at least once a year – to work on particular themes and tasks. BSRLM currently lists five active WGs:

I. Building and Sustaining Active Research Collaborations with Teachers of Mathematics
II. Critical Mathematics Education
III. Early Years and Primary Mathematics (EYPM)
IV. History in the Mathematics Curriculum (CME)
V. Using Statistics in Mathematics Education Research

and three past WGs:

I. Trigonometry
II. Geometry
III. Sustainability and Mathematics Education

Although the BSRLM website lists eight past and present WGs, during the period of the review, 38 different WGs met amounting to 91 sessions. It is perhaps worth noting that 17 of these WGs only met on one occasion (and of these, only one – *Lesson Study in Research and CPD in Mathematics Education* (Archer et al., 2013) – produced a WGIP). We were surprised to find that only 4% of the corpus (34 Proceedings) were reports from WGs.[7] Further analysis showed that these 34 WGIPs came from 14 separate WGs with the Trigonometry and Geometry WGs publishing five and eight WGIPs respectively. WG sessions are less likely to be subsequently written up than other sessions; while 56% of all sessions between 2003-2017 translated into Proceedings, only 37% of WG sessions were published.

Where sustained, WGs have the potential to provide a springboard to substantial future work and developments, both within and beyond BSRLM. For example, Jones and colleagues used discussion within the Geometry WG (Jones et al., 2009) to develop and disseminate a Professional Development programme – produced in conjunction with NCETM – for teachers using GeoGebra (Lavicza et al., 2010). Further, other members of the Geometry WG have since published a review of

[7] Some WGs have produced notes available via the website rather than published Proceedings. Further, members of WGs may disseminate work arising from WGs in standard research sessions / Proceedings, neither of which are captured here.

geometry education from the early years through to post-compulsory education, identifying seven core threads within the literature (Sinclair et al., 2016).

As an interesting example, the *Building and Sustaining Active Research Collaborations with Teachers of Mathematics* WG has not only produced academic writing beyond the WG in their survey of teacher collaboration internationally (Robutti, et al., 2016) but have also been instrumental in setting up the BSRLM Blog (Clark-Wilson & Adams, 2016),[8] opening up the work of not only the WG, but also BSRLM, and the field of mathematics education, to a wider audience. These are clearly examples – and others exist – of the potential strength of the WGs. We explore the implications of this and our findings further in Section 8.

4.2.2 Methodology and method

Within the Proceedings reporting empirical research, 65% generated qualitative data, with mixed methods and quantitative methods accounting for 20% and 14% respectively.[9] The international picture here is somewhat similar. Hanna & Sidoli's (2002) review suggests that approximately 15% of studies used quantitative methods with the remaining 85% using a range of qualitative approaches (perhaps indicative of the time period, studies were only categorised by one approach, hence no figures being available for mixed methods research). Hart et al. (2009) found that 50% of the studies they examined used qualitative methods only, 21% used quantitative methods only, and 29% used mixed qualitative and quantitative methods in various ways. The higher proportion of mixed methods studies here is most likely to be indicative of the different natures of BSRLM Day Conferences – which attract many students and ECRs and often report work in progress – and the prestigious journals included in Hart et al.'s study which are more likely to represent the end point of larger projects and are potentially reports of larger and/or funded projects which may have the capacity and expertise to employ a wider range of research approaches.

Case-studies accounted for 31% of the Proceedings reporting empirical research. As we shall see in Section 7 (Teachers and Teacher Development) this is perhaps unsurprising given the ease of access of those working in HEIs and particularly ITT to the conditions for a case-study. In relation to research methods, the four most frequently adopted methods were interviews, including clinical interviews (interviews were used in 31% of the corpus, rising to 43% of the Proceedings reporting empirical research), observations and questionnaires/surveys (both used in 21% of the Proceedings reporting empirical research), and the use of video, including video-recall interviews (used in 16% of the Proceedings reporting empirical research). The use of each method was far from homogeneous. For example, although 43% of the Proceedings reporting empirical research used interviews, this ranged from the interview being used as the sole method in 18% of these Proceedings, to the interview being used alongside the full range of other methods. Further, the number and purpose of the interviews conducted varied enormously; for example, Trubridge & Graham (2007) conducted 44 interviews as part of their longitudinal study into the *Collaborative Connected Classroom* model for CPD for secondary mathematics teachers, allowing them to draw on the standpoints of multiple teachers working in different ways across a sustained time-period. At the other end of the scale,

[8] All BSRLM Blogs are available at: http://www.bsrlm.org.uk/category/blogs/

[9] Note that throughout this review percentages are rounded to whole numbers and as such may not sum to 100%.

Tasara's (2017) study illustrates how a very limited sample (in this case of one teacher) allowed for comprehensive transcription and discourse analysis, focussing in-depth on word use, visual mediators, routines, and endorsed narratives.

The breakdown of research methods in the BSRLM Proceedings is partially in-line with Hanna & Sidoli's (2002) earlier ESM review which found interviews to be the most common research method, used in 20% of the empirical studies they examined. The predominance of these research methods – and in many cases, small-scale studies – corresponds with Inglis & Foster's (2018) finding of a sharp decline in mathematics education of research employing experimental designs which they posit may be due to a migration of such studies to psychology. Of note, we were surprised to find that while research methods – and to an extent methodology – were well-documented, almost half of the Proceedings reporting empirical research did not specify the analytic approach used with vague phrases such as "the data were analysed". This perhaps represents an area for future development.

4.2.3 Video as method

As noted above, 16% of the Proceedings reporting empirical research used video-recording as a research method. This is a significant change from earlier reviews, most likely indicative of technological changes and the subsequent increased ease of using the method. Across the time-phases, we found a slight increase in the use of this method from 2003-2007 to the subsequent two phases. In earlier years, video recording was not mentioned as a research method in Hanna & Sidoli's (2002) review which covers papers published from 1990-1998, and is only mentioned once in Nickson's (2003) previous BSRLM Proceedings analysis (covering the Proceedings from 1995-2002) in relation to the audio/video recording of classroom situations alongside a wide-range of other methods in larger ethnographic studies.

As with interview studies, the use of video as a research method / tool is diverse, with six usages extracted from the Proceedings. Video was used:

I. In place of, or in addition to, traditional observations, allowing for rigour in larger studies with multiple research team members (Mooney, Fletcher & Jones, 2003)[10], providing opportunities to revisit observations conducted over an extended time period (Hewitt & Coles, 2017), or allowing the teacher, as researcher, to collate observations of themselves in action research approaches (Pendlington, 2004);

II. For repeated viewing by multiple members of research teams to develop or test analytic frameworks (Huckstep, Rowland & Thwaites, 2003; Smith & Piggott, 2009) or to explore the application of a range of frameworks to observational data (Watson, 2006);

III. In comparative studies, be this across classrooms (Coles, 2008), between institutions (Karaagaç, 2004) or between countries (Andrews, Hatch & Sayers, 2005);

IV. To focus on behaviours difficult or impossible to capture through audio-recording such as gesture and gaze (Wylie, 2007); ideas which "are complicated by the partially internal nature of these constructs" (Barclay, 2018, p.3) such as mathematical awareness (Brown, 2017), or

[10] All citations are examples; other Proceedings fall into these categories but are too numerous to list here.

silences which could have a range of rationale behind them, such as in Andrews, Ingram & Pitt's (2016) study of teachers use of deliberate pauses;

V. In a developing sphere, capturing pupils' work using software packages such as *Autograph* (Gibbs, 2005) or other similar interactive hardware such as a multi-touch table (Nikolakopoulou, 2016) allowing the simultaneous capture of on-screen/table work and pupil interaction;

VI. As the intended outcome or subject of research, as seen in Griffiths et al.'s, (2006) DVD project for parents supporting them in helping children under five learn to count and in Coles' (2010) Proceeding exploring the use of video in teacher professional development.

In addition to the above categories where the video *was* the data, video has been used in video-stimulated interviews. These have been used across the time period of the review, yet their usage is very limited with three of the four studies being part of *The Knowledge Quartet* suite of work (Turner, 2007; Aldalan & Rowland, 2014; Thwaites, Huckstep & Rowland, 2005) and only one sitting outside of this, in which video-stimulated interviews were used to support lecturers' recall of tutorials in HE mathematics (Jooganah & Williams, 2010). In all studies, video is not the data collection method itself, but feeds into the data collection method of a video stimulated / recall interview allowing the interviewee to comment on selected aspects of their recently videoed teaching. This interview is audio-recorded and transcribed and it is this transcription that forms the 'data' of the study. There is some evidence from these studies that the video-stimulated interview facilitates deeper reflection and explanation, particularly with trainee teachers who are likely to be focussed on a multitude of other aspects of their teaching quite possibly outside of the focus of the researcher. However, along with other rarely seen approaches such as Peters' (2010) use of eye-tracking technology in comparing the fixation times of expert and novice mathematicians, this is still a relatively novel method – at least within BSRLM Proceedings – and it waits to be seen how the use of such methods develops.

4.3 Study populations

The Proceedings address mathematics education across all age phases (Figure 2). Phases attracting the largest number of the Proceedings are secondary (35%) and primary (26%). Despite the relatively strong focus on primary mathematics, EYFS is the focus of less than 1% of the Proceedings. These statistics are somewhat similar to the ESM review (Hanna & Sidoli, 2002); here there were only four categories of Primary (43%), Secondary (47%), Tertiary (9%) and Adult (1%). It should be noted that 'Primary' was deemed to be from the age of five; there was no category – and one would therefore assume no papers falling into the category – of Early Years.

The proportions in the present review represent an improvement on the findings of Nickson's (2003) review which found that the majority of the Proceedings came from the secondary sector, to the extent she called for a stronger primary focus in future BSRLM work. This call has been partially heeded; we do now see a greater focus on the primary phase and secondary, although the largest sector, is no longer the majority. Moving forward, we would hope to see a greater balancing. It may be that groups such as the recently established Early Years and Primary Mathematics (EYPM) WG go some way towards achieving this. Each of these educational phases is considered in detail – examining the topics and foci of the research conducted within each phase – in Section 5.

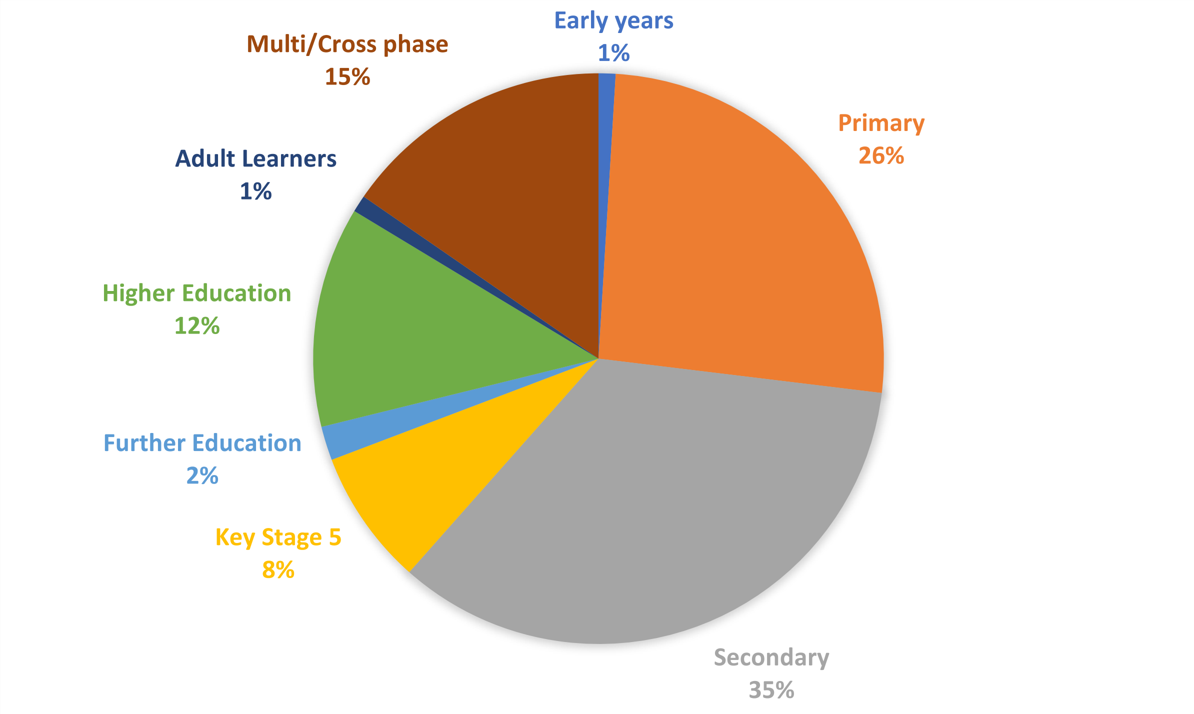


Figure 2: Age-phase representation across the Proceedings

Beyond the educational phases, we also analysed the target study population of each Proceeding. Unsurprisingly, almost half focussed on pupils/students engaged in mathematical activities. Trainee teachers and teachers accounted for the next two most frequently studied populations at 18% and 12% respectively; as a repeated theme across our coding, we look in-depth at the Proceedings examining Teachers and Teacher Development in Section 7.

Two study populations were conspicuous in being almost absent. Classroom teaching assistants (TAs) were very infrequently the focus of studies, only being designated as the study population in four of the Proceedings. Of these, only two, both by Houssart (2011, 2012), explored TAs' practices in class and individual/small-group situations, finding a high degree of intuitive, adaptive work, with TAs well-placed to respond to, and knowledgeable of, individual learners' needs. It was noted in these studies that although we saw something of TAs' pedagogic content knowledge in action, we cannot ascertain the extent or nature of their mathematical content knowledge or the impact of this. Crisp (2013) began to engage with this in looking at the impact of a '*Subject knowledge and professional practice in primary schools*' module on TAs' practice, but concluded there is still much to know and different data collection approaches may be needed. Spencer & Edwards (2011) did propose a different way of looking at TA practice through tracking TA and teacher movements around the secondary mathematics classroom although this does not yet appear to have been developed further.

The second specified group given limited attention (less than 1% of the Proceedings) was learners with SEND or mathematical difficulties. Most of the Proceedings in this category focus on the identification – and in all raise the issues and difficulties of identification – of learners with specific mathematical difficulties (Gifford, 2005; Voutsina & Ismail, 2007; Gifford & Rockliffe, 2008). Where there exists a significant gap in coverage within the Proceedings is in studies identifying or evaluating approaches to working with learners with mathematical difficulties in the classroom. Just one Proceeding (Zerafa, 2014) – looking at the use of the *Catch Up Numeracy* intervention programme – begins to address this area, but having been conducted in Malta and translated into Maltese raises questions around generalisability of findings.

4.4 Foci of the Proceedings and mathematical topics covered

As noted in Section 3, we categorised and examined the content of the Proceedings in a number of ways. At a general level, grouping by MESC classification allowed us to see that broadly, the majority of the Proceedings (73%) fell into three classifications:

i. **MESC classification C:** Psychology of mathematics education. Research in mathematics education. Social aspects. (29%)
ii. **MESC classification D:** Education and instruction in mathematics (24%)
iii. **MESC classification B:** Educational policy and educational system (Educational research, educational reforms, pilot projects, official documents, syllabuses) (19%)

Given the nature and foci of BSRLM – research and learning in mathematics – these are unsurprising. The proportions by which our thematic groups were applied to the three keywords given or assigned to each Proceeding provide more detail as to their focus (Table 2).

Thematic Group	Proportion
Teachers	16%
Mathematical topics	12%
Affect	9%
Classroom Approaches	8%
Pedagogic Tools	8%
Mathematical thinking	7%
Curriculum & Pedagogy	7%
Educational Research	6%
Classroom talk & Interaction	6%
Assessment & Accountability	5%
Social Context	5%
Phases	5%
Cognition	5%
Developmental trajectories & SEND	3%

Table 2: Proportion of keywords assigned to each thematic category

Again, the two most common thematic groups – Teachers and Mathematical Topics – are perhaps not surprising for a society that addresses learning in mathematics. A significant proportion of the Proceedings allocated the thematic group of 'Teachers' focused on Initial Teacher Training or CPD which seems to be an encouraging finding given that Nickson (2003) noted a need to extend and further nurture research-involved CPD in her previous review. Given that 'Teachers' accounted for the greatest proportion of thematic group application to the keywords, we examine the Proceedings within this subset in detail in Section 7: Teachers and Teacher Development. Other thematic groups, such as 'Affect' are picked up further in our discussion of age-phases (see Sections 5.2.2 and 5.3.1). We also found that the policy climate – and in particular the significant number of policy changes seen within mathematics education – cut across a number of thematic groups and hence Proceedings. As such, we look in Section 6 at 'Policy, Curriculum and Pedagogy' with a particular eye to the impact of various milestones in policy.

It is interesting to note that although our thematic groups arose from coding the keywords – i.e. they were developed a posteriori rather than a priori – there is a strong synergy between our thematic groups, the CERME Thematic Working Groups (Dreyfus et al., 2018), and the topics/categories arising from the reviews of ESM/JRME (Inglis & Foster, 2018) and ESM (Hanna, & Sidoli, 2002) (Table 3). The categories featuring most strongly in Hanna & Sidoli's (2002) were cognitive, affective and pedagogical, broadly similar to our findings. We note that educational research was not a category in their 2002 review yet features in both 2018 reviews as well as in our BSRLM Proceedings review, a development we find particularly encouraging.

Review	Groups / Categories / Topics (all presented in alphabetical order)
CERME Thematic Working Groups	Affect; Algebra; Classroom practice; Comparative studies; Creativity & talent; Diversity; Early years; Geometry; HE; History & mathematics education; Language; Modelling; Number sense; Probability & statistics; Proof; Teacher education & professional development; Technology; Theory & educational research
Inglis & Foster's (2018) ESM/JRME topics	Addition & subtraction; Analysis; Constructivism; Curriculum (especially reform); Didactical theories; Discussion, reflections & essays; Dynamic geometry and visualization; Equity; Euclidean geometry; Experimental designs; Formal analyses; Gender; History and obituaries; Mathematics education around the world; Multilingual learners; Novel assessment; Observations of classroom discussion; Problem solving; Proof and argumentation; Quantitative assessment of reasoning; Rational numbers; School algebra; Semiotics and embodied cognition; Sociocultural theory; Spatial reasoning; Statistics and probability; Teachers' knowledge and beliefs; Teaching approaches
Hanna & Sidoli's (2002) ESM 1990-1998 review categories	Affective issues; Cognitive issues; Epistemological issues; Gender & ethnicity; Historical analysis; Imagery / visualization; Language; Pedagogical & didactical; Reform & curricular issues; Social & cultural issues; Technology

Table 3: Categories used/arising from other reviews of the mathematics education field

In relation to 'Mathematical Topic', Table 4 gives the frequency of keywords within this thematic group. We look at these in relation to age-phases in further detail in Section 5: Phases and Topics in Mathematics Education. Both algebra and geometry represent specific topics of interest covering 14% of the topics each. Our findings are entirely consistent with the three highest mathematical topic areas found in ESM papers from 1970-1999: Geometry or space (24%), Algebra (10%) and Number or calculation (12%). There is also continuity here between Nickson's review of the BSRLM

Proceedings from 1995-2002 and our 2003-2017 review with the same topics highlighted as featuring strongly. As a point of change, it is interesting to note that over a third of the Proceedings examining algebra and a half of the Proceedings examining geometry also focused on technologies (such as the use of dynamic geometry software). This represents a significant improvement on the 'limited focus' noted in the 1995-2002 review.

Keywords in 'Topic'	Occurrence
Algebraic reasoning	14%
Geometry	14%
Calculus	7%
Fractions	6%
Functions	6%
Modelling (mathematical)	6%
Arithmetic	6%
Calculation	5%
Numeracy	4%
Number	4%
Multiplicative reasoning	3%
Trigonometry	3%
Statistics	3%
Mental mathematics	2%
Probability	2%
Written methods	2%
Division	2%
Equivalence	2%
Number sense	2%
Proportional reasoning	2%
Equations	1%
Mechanics	1%
Randomness	1%
Classification	1%
Area	<1%
Number theory	<1%
Set theory	<1%
Subtraction	<1%

Table 4: Occurrence of keywords in the mathematical topic thematic group

Following this overview of the full corpus, we turn in the next three sections to an in-depth analysis of the key areas arising in this survey.

5 Phases and Topics in Mathematics Education

This section explores the various age phases, and the research foci and mathematical topics covered within these. In examining the coverage by phase, it is worth comparing with the overall topic coverage discussed in Section 4.4. In most cases – with the exception of cross-phase research – the overall pattern of thematic group coverage for the corpus was the same as for each age-phase.

We were interested to note that certain specific concepts came up across the primary and secondary phases, hence a section within each devoted to issues of affect. An interesting aspect of the discussion to follow is the development of the work beyond that outlined in the BSRLM Proceedings, demonstrating a reaction to external policy changes. Examples of this include the ICCAMS project, Underground Mathematics project, Cornerstone Maths project, and reactions to the introduction of the NNS in English primary schools.

5.1 Early Years

Less than 1% of the Proceedings address mathematics teaching and learning in the EYFS, with two main topics identified. The first of these is the development of young children's number understanding; the second is CPD for early years practitioners. The Proceedings addressing CPD for early years practitioners promote the value of collaborative practitioner partnerships; this is addressed in Section 7. This section summarises the findings of the small number of the Proceedings relating to young children's number understanding.

Gifford's (2015) critique of the research base for the number curriculum for 3 to 7 year olds identified a lack of research support for some aspects of number learning goals in early years curricula, for example counting back to subtract and solving problems involving doubling and halving. Gifford concluded that research strongly supports a priority in early years on the development of a feel for number and of relationships between numbers. In particular she reported that research shows that an understanding of cardinality, comparison of relative size and the development of part-whole concepts are predictive of later achievement and should be the priority for early years mathematics. Since Gifford's Proceeding was published, Ofsted commissioned a report into the Reception curriculum (for ages 4 to 5) in England entitled *Bold beginnings: The Reception curriculum in a sample of good and outstanding primary schools* (Ofsted, 2017) which Gifford has responded to through the BSRLM Blog (See Section 4.2.1, p.16 of this review for discussion of the development of the Blog) in which she notes that, while Ofsted's intention was creditable, the report may have the unintended effect of formalising teaching in Reception and narrowing the mathematics curriculum.[11]

The development of cardinality, amongst other counting principles, is the focus for Gray's (2015) small scale study of nursery children. Gray's study focused on children who were all EAL learners and who entered nursery education at below age-related expectations. Gray found no typical

[11] See 'Bold Beginnings: a missed opportunity?' available at http://www.bsrlm.org.uk/bold-beginnings-missed-opportunity/. For further discussion of the impact of Bold Beginning, see also the special edition of FORUM which provides a response to this report (Marks & Yarker, 2018), including a response to the mathematics element of Bold Beginnings (Briggs, 2018).

developmental trajectory for the development of counting skills and noted that these children's understanding of cardinality in particular was not secure by the end of the academic year.

The development of counting in home-based learning is addressed by Griffiths and colleagues (2006). Their conclusion was that education professionals should celebrate and encourage the enthusiasm of families for supporting young children's counting and should value the highly personalised nature of the activities in which young children can engage at home.

The very small proportion of the Proceedings addressing the development of mathematics understanding in the early years foundation stage was noted in our interim review and gives rise to recommendations for developing closer links with EYFS professional networks.

5.2 The Primary phase

Analysis of thematic groups for the Proceedings covering the primary phase revealed a significant focus on issues relating to teachers (including teacher training, teachers' professional learning and subject knowledge). The second largest key word group related to mathematical topics. Within the 'topic' group, calculation, reasoning and understanding of number featured strongly. The third largest group was 'affect', reflecting a strong interest in primary pupils' beliefs, perceptions and attitudes towards mathematics and themselves as learners.

Issues relating to teacher development and subject knowledge are addressed in Section 7; this section addresses themes within the primary phase where *pupils/students engaging in mathematics learning* are the study population.

5.2.1 Calculation and the impact of policy

The role of BSRLM in analysing and critiquing policy and its impact on pupil learning is evident in the Proceedings addressing the impact of the introduction of the NNS in English primary schools in 1999 (see Section 6 for a fuller analysis of policy). The case of the implementation of the NNS demonstrates the significant role policy can play in instigating major curriculum change, both in learning and teaching.

In an early evaluation of the impact of the NNS on pupils in Year 4, Brown, Askew & Millet (2003) identified improvements in some aspects of number understanding, place value and mental calculation for addition and subtraction but noted that this came with decreased attainment in multiplication, division and the application of mathematics. Further, in relation to specific learners, the authors note with concern that lower attaining pupils appeared to gain little from whole class teaching sessions. This same concern was raised by Kyriacou & Goulding (2004) who reported early findings from a DfES funded systematic review of the daily mathematics lesson in the context of the NNS. Here, the authors identified that the focus on maintaining good pace and strict time management in whole class interaction was at the expense of developing reflective and strategic thinking which was detrimental for lower-attaining pupils. Attention to the needs of lower-attainers in primary mathematics is revisited in the following subsection.

Borthwick & Harcourt-Heath contributed five connected Proceedings (2007, 2010, 2012, 2014, 2016) analysing Year 5 (10-11 year-olds) children's use of and success with a range of calculation strategies.

Their data show an increase in the proportion of pupils able to answer age-related, de-contextualized calculation questions in all four operations over the period studied. The grid method has consistently been the most used strategy for multiplication, also consistently suffering from frequent computation errors in its use. Up to 2014, subtraction methods based on number lines, and division methods based on chunking were most commonly used and with greater accuracy than the use of standard algorithms. This is conversant with the flexibility in strategy choice advocated by the NNS. In relation to division, this supports the findings of Anghileri (2005) who also noted that the use of informal chunking supported improvements in accuracy with division in Year 5 pupils. Two years after the introduction of the new National Curriculum (NC) with its increased emphasis on the use of standard algorithms, Borthwick & Harcourt-Heath's (2016) data show a steep rise in the proportion of pupils attempting and being successful with standard written methods for subtraction and division. The authors register concerns however about the errors made in attempts at written methods and possible gaps in pupils' conceptual understanding that these errors reveal. The longer-term impact of the 2014 NC focus on pupil use of and accuracy with standard written algorithms is yet to be established.

Analysis of policy emerged again in more recent years with a small number of the Proceedings addressing a mastery approach to teaching in primary mathematics (see Section 6.3: *'Mastering the curriculum': Pedagogy and policy in mathematics education today,* for a more detailed discussion of mastery across the Proceedings). While both Clarke (2017) and Spencer & Fielding (2015) explore the use of the bar method and note is potential usefulness in supporting reasoning about calculations, Duckworth et al. (2015) question whether a mastery approach offers anything substantially different in its aims for teacher pedagogy and pupil learning. This represents an area of ongoing interest.

5.2.2 Affect, classroom practice and lower-attaining pupils in primary mathematics

Although rarely evident in the titles or key words, we note a recurring concern within the primary phase Proceedings regarding mathematics learning for lower attaining-pupils. This theme is found in the Proceedings addressing affective issues, but also in some of those addressing classroom practices and the development of number understanding. Where affective issues are addressed, self-belief and attitudes towards mathematics among lower-attainers' raise concerns. Separately, Ashby (2009) and Whitebread & Chiu (2003) note that lower-attainers draw connections between being 'good at maths' and being clever, with the deterministic view of mathematics ability expressed eloquently and arrestingly by one pupil: "mathematics belongs to people who are really smart" (Whitebread & Chiu 2003, p.65). For Ashby, lower-achievers' low self-belief and resignation to failure was compounded by a difficulty in seeing the purpose of mathematical activities and in interpreting mathematical language.

Concern for the welfare and progress of lower attainers is addressed by Marks (2011a, 2011b, 2012) through her analysis of the impact of ability-grouping practices. Marks finds that these practices reinforced pupils' belief of their own capacity and that pupils were aware of the ability judgements underpinning grouping practices regardless of the labels applied. Moreover, pupils in the lowest set were aware of, and felt disadvantaged by, the differential opportunities in teaching approach and access to resources that they were afforded compared to learners in other sets. They were also aware of the low expectations of SATs that their teachers had of them. BSRLM provided a grounding for Marks' work on ability grouping to have broader impact; the Proceedings led to an output in *RME* (Marks, 2014) and her work has been cited by several significant authors (e.g. Anthony and Hunter 2017; Archer et al., 2018) as well as being referenced in a recent DfE policy document (Boylan et al., 2016).

More optimistically, Pendlington (2006) reports on a project with lower-achievers who initially displayed powerlessness in relation to their mathematics learning. Her approach valued struggle, choice, autonomy, questioning and reflection. Pupils engaged in collaborative exploratory tasks, revealing to them what they did not understand; this prepared them for more active engagement in the more structured teaching tasks that followed as they knew what they needed to understand. She reports their increasing tendency to challenge, raise questions and negotiate meaning.

Together, the Proceedings discussed above reflect a persisting concern in relation to the extent to which organisational and pedagogic structures sufficiently support lower-attainers in mathematics; in the light of this it is of interest that low-attainers are so infrequently the overt study population.

5.3 The Secondary phase

246 of the Proceedings focussed on secondary education. An analysis of thematic groups (see Table 5) indicated that these tended to focus on teachers, topics (algebraic reasoning and geometry were dominant in this respect), classroom approaches, affect, pedagogic tools, and mathematical thinking.

Keyword Thematic Group	Frequency	%
Teachers	125	17
Topics	88	12
Classroom Approaches	74	10
Affect	68	9
Pedagogic Tools	65	9
Mathematical thinking	60	8
Classroom talk & Interaction	51	7
Social Context	40	6
Assessment & Accountability	35	5
Cognition	32	4
Curriculum & Pedagogy	28	4
Educational Research (methodology & approach)	23	3
Developmental Trajectories and SEND	18	3
Phases	10	1

Table 5: Thematic group application to Proceedings covering the secondary phase[12]

[12] Note that frequencies sum to more than the total number of the Proceedings as many fell into more than one thematic group

5.3.1 Affect and boosting mathematics participation

There have been some significant Proceedings in relation to affect in secondary level education. *'I would rather die': Attitudes of 16-year-olds towards their future participation in mathematics* by Brown, Brown, & Bibby (2007) is a prime example. This Proceeding focussed upon a sample of 1,997 Year 11 pupils from seventeen schools who completed a questionnaire on their experiences of examination systems and their attitudes towards mathematics. This Proceeding provided a detailed insight into the reasons for continuing with study of mathematics post-16: enjoyment, usefulness for a career, confidence of future success, and the importance of mathematics. It also highlighted reasons for not continuing to study mathematics post-16: perceived difficulty, dislike, expectation that it would not be required for future studies or career, and perception that it is boring. It was concluded that placing a renewed focus on supporting females and those with B grades to continue with mathematics, highlighting the importance of mathematics, and enhancing pupil confidence in the subject were a means of increasing Post-16 participation. This Proceeding was the pre-cursor to a journal article with the same title published in the inaugural issue of *Research in Mathematics Education* in 2008. This work was cited 32 times at the time of writing and has contributed to recent discussion papers on provision of post-16 mathematics (e.g. Hodgen, Marks & Pepper, 2013).

This theme of boosting motivation to enhance participation in Post-16 mathematics continued in earnest, with two significant Proceedings in this area in 2009. Bond, Green, & Jaworski (2009) reported upon interventions carried out in four schools to encourage high-attaining pupils to continue studying mathematics beyond the age of sixteen. Each intervention was different in nature and the approaches included acceleration, enrichment, early commencement of A-Level study, and the provision of the Free Standing Mathematics Qualification. Meanwhile, Hodgen et al. (2009) presented their work on *Lower secondary school students' attitudes to mathematics: Evidence from a large-scale survey in England*, a large-scale survey of 1,422 11-14 year olds in ten schools in England. They noted that pupil attitudes towards mathematics declined in positivity as the pupils got older, with this being more pronounced amongst girls than boys. The proportion of pupils indicating an intention to study mathematics beyond GCSE also dropped as age increased. This drop was more pronounced amongst girls than boys. The authors were surprised, however, that the vast majority of those surveyed (>85% in each year group) responded that hard work was more important than natural ability in determining success in mathematics.

This collection of research appears to have formed an early platform for further work in this field, leading to impactful journal articles and policy documents. For example, Hodgen and Brown were heavily involved in the EEF-funded report *Evidence for Review of Mathematics Teaching: Improving Mathematics in Key Stages Two and Three* (Hodgen et al. 2018). Hodgen also led a 2013 Nuffield Foundation sponsored report entitled *Towards universal participation in post-16 mathematics: lessons from high-performing countries* (Hodgen, Marks & Pepper, 2013). Work of this nature, which was presented in its initial stages at BSRLM, could be linked to enhancements made in recent years to improve the uptake of post-16 mathematics, particularly the provision and proliferation of Core Maths which was introduced from September 2014 onwards.

5.3.2 Technology and representations

The potential for technology to enhance teaching and learning of secondary mathematics is a consistent thread throughout the BSRLM proceedings. A key example is the Cornerstone Maths project discussed by Clark-Wilson, Hoyles, & Noss (2013). This project, in its pilot phase at that point, aimed to embed dynamic mathematical technology (DMT) into lower secondary mathematics curriculum units, with a particular emphasis on big mathematical ideas, linking key mathematical representations, and facilitating structured problem solving activities. This project was a collaboration between US and UK institutions which aimed to provide teachers with the materials and professional development to utilise this technology effectively in the classroom. Their Proceeding, *Cornerstone Maths: Designing for Scale*, reported on the progress made at that point and plans to scale this project to over 100 schools. This not only gave an insight into educational innovations utilising technology but also analysed the means by which scaling a design-based research methodology can be effectively completed.

Presently, this project appears to be making strong progress as Clark-Wilson & Hoyles (2018) published an article in ESM reporting outcomes from implementation of the technology in 42 London secondary schools involving 111 teachers in the period 2014-2017. They also released a report in April 2017, supported by the Nuffield Foundation, entitled *Dynamic Digital Technologies for Dynamic Mathematics*. They indicate that, within the next stages of the project, they plan to measure the impact of this project on teachers' Mathematical Pedagogical Technology Knowledge (MPTK) in the near future (Clark-Wilson & Hoyles 2018).

5.3.3 Mathematical topics and classroom approaches

From the 246 Proceedings specific to the secondary level, algebraic reasoning was a keyword for 19 of these, while geometry was a keyword 17 times. The regular appearance of these topics is consistent with the findings of the previous BSRLM review. Research related to these two topics regularly looked abroad for guidance on how to teach these topics effectively. Jones, Fujita, & Ding (2005), responding to questions regarding the quality of teaching of geometry in the UK, analysed how expert teachers in high performing countries (according TIMSS) such as Japan and China conducted lessons in this topic at lower secondary level. They concluded that there are common principles for effective teaching of geometry but could not establish a specific set of teaching methods that would be expected to impact learning significantly regardless of the setting. Ding & Jones (2006) built on this earlier work. They utilised observation data from Year 9 lessons in Shanghai to inform understandings of effective teaching of geometry. It was determined that teachers in this region develop pupils' thinking as they transition to deductive geometry by utilising strategies which try to reinforce visual and deductive approaches. This is achieved through three strategies: learning and frequently reviewing theorems; gradually drawing complicated figures while presenting proof problems; and separating complicated figures into a series of basic figures. This study appeared to demonstrate an evolution in understanding of the methods applied by teachers in this region and offers an early insight into how teaching in this region started to influence approaches adopted recently in the UK.

Analysis of approaches to teaching Algebra at secondary level has been prominent at BSRLM. The *Increasing Competence and Confidence in Algebra and Multiplicative Structures* (ICCAMS) project has played a central role in this respect, accounting for seven of the Proceedings between 2008 and 2017. Hodgen and Küchemann were key contributors to these Proceedings, along with Brown and Coe. A prime example of this work was entitled *Models and representations for the learning of multiplicative reasoning: Making sense using the Double Number Line* (Küchemann, Hodgen & Brown, 2011). This explored the value of utilising the Double Number Line (DNL) to aid the development of pupils' multiplicative reasoning. They concluded, upon analysing the introduction of the DNL to a high-attaining Year 8 group, that this model of scaling should aid the development of pupils' multiplicative understanding but would require time and effort in order for pupils to understand and utilise the model effectively. The ICCAMS project ended in the summer of 2018 but there are a range of related resources available and plans to provide professional development to teachers based on the progress made in understanding through this research. An EEF-funded evaluation of the project is currently ongoing and is expected to be released in 2019.[13]

5.4 Further Education and Higher Education

159 of the Proceedings focussed on FE and HE settings. Analysis of the thematic groups within these Proceedings indicates an emphasis on teachers and lecturers, affect, assessment & accountability, curriculum & pedagogy, and topics such as calculus and functions. A prime example of research examining the FE/HE settings specifically was presented by Jooganah & Williams (2010). They investigated the development of advanced mathematical thinking among students transitioning from school/college to university study. Through the use of ethnographic methods, including observations and interviews with lecturers and students, they concluded that cognitive conflicts can result due to the contradictory mathematical practices applied in schools and universities. They recommended that exploring the means by which consistency between schools/colleges and university approaches to teaching and learning mathematics may be established could alleviate some of the issues highlighted.

Considerations of the experiences of students in HE settings were also present in a Proceeding by Macrae, Brown, & Bartholomew (2003). They utilised quantitative data, which mainly consisted of examination performance, and qualitative data in the form of interviews with six students who were deemed to be 'at risk' of failure in their second year of single honours mathematics studies at an English HE institution. From their analysis, they determined that these 'at risk' students struggled to adapt effectively to the demands of their HE studies, particularly the increase in independent learning. They indicated that pre-HE learning experiences included greater levels of structure within which students would be "spoon-fed" for longer. Recommendations to remedy this included the provision of compulsory tutorials and homework assignments so that 'at risk' students could be identified early in their HE studies. This would be followed by appropriate intervention.

5.4.1 Underground Mathematics

The Underground Mathematics project, funded by the DfE, has played a significant role in post-16 mathematics classroom practice development since its launch in February 2016. The extensive teacher resources created through this project have been designed to facilitate students to engage

[13] See: https://educationendowmentfoundation.org.uk/projects-and-evaluation/projects/increasing-competence-and-confidence-in-algebra-and-multiplicative-structur/

with a rich body of mathematical knowledge through problem solving, mathematical thinking, and dialogue (Figure 3). This expansive project, which grew from the Cambridge Mathematics Education Project (CMEP), commenced in October 2012, and was reported upon by Rostovtseva & Walker (2015). They outlined methods for evaluating implementation of these resources and the associated learning environments using case-study research across a range of KS5 classrooms in England.

Consideration of the means by which projects of this nature can be assessed for impact on learning and how the characteristics which define Underground Mathematics manifest in the classroom continued in subsequent BSRLM conferences. Rauch et al.'s (2017) Proceeding entitled *Studying the link between classroom dialogue and the implementation of rich tasks in post-16 mathematics with Underground Mathematics* discussed the early stages of the implementation of resources designed to support teaching and learning of mathematics through this project as well as the methods implemented in a pilot study which aimed to determine the impact of this project as it progressed. Initial findings indicated that the use of teaching resources fostered rich dialogue between students as they engaged in productive struggle and enhanced their understanding of mathematical concepts explored. Funding for this project provided by the DfE was discontinued in September 2017 but resources are still available on the Underground Mathematics website.[14]

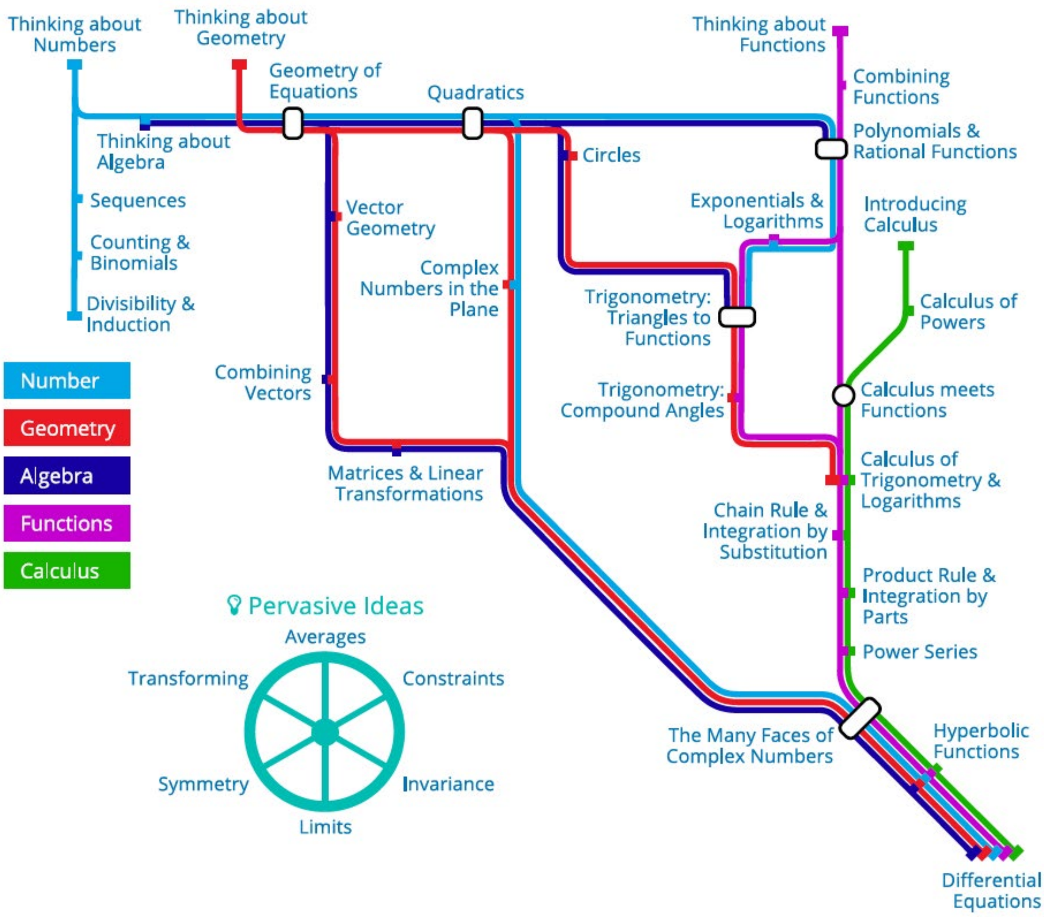


Figure 3: Underground Mathematics: Rich resources for teaching A level mathematics

[14] See: https://undergroundmathematics.org/

5.5 Cross-phase research: developing partnerships between practitioners and academics

The Proceedings addressing issues of cross-phase significance include a much smaller proportion of empirical studies than other phase categories. Instead, reports (including those from working groups) and discussion papers demonstrate the value of BSRLM as a site for the development of ideas across phases and a space for BSRLM members to develop their thinking and practice in particular spheres.

One recurring theme addressed through both discussion papers and Working Group activity is the relationship between research and practice. Two related topics within this theme are collaborative partnerships between teachers and researchers, and the impact of research on teaching and teacher development. Across the review period this theme has remained an area of interest; the content of the Proceedings reveals that the challenges encompassed remain unresolved.

The Proceedings addressing partnership between teachers and researchers are underpinned by the belief that such partnership is an essential component of achieving impact on teaching. For example, Jaworski (2003) details examples of partnership research resulting in co-learning and then considers the extent to which education design research lends itself to the development of such partnerships. She notes that whilst design research involves teachers, research reports frequently leave the precise nature of the partnership unclear. As a result, she questions the extent to which shared ownership of the intervention and the arising theory and practice recommendations is always achieved.

Watson (2010) and Clark-Wilson & Wake (2016b) support the argument for collaboration between teachers and researchers, but also identify challenges and barriers to the development of such partnerships. These include the differing understandings that teachers and researchers may hold about what constitutes meaningful and ‘good’ research, difference in the goals of partners involved, lack of alignment between institutional constraints and research goals, and access to schools by researchers to develop partnerships. Clark-Wilson and Wake (2016a) note that amongst working group participants, collaborative projects were nearly always stimulated by researchers. This is of interest given Watson’s (2010) call for research to more consistently build on the concerns and issues faced by teachers and to draw on practitioner knowledge.

The impact that research can have on practice was addressed by Kyriacou & Issitt (2008, p.39) who responded to the growing drive towards “evidence-based education” through the use of systematic reviews of research evidence. The authors concluded that the “inconsistent and equivocal” nature of research evidence drawn on in such reviews, together with the need to adapt recommendations to individual, highly varied educational contexts means that systematic reviews of the type currently in vogue through the work of bodies such as the EEF is unlikely to lead to the developments in teaching that the findings of such reviews recommend.

Underpinning the discussion reported by Joubert et al. (2012) was a recognition that research needs to be more accessible to teachers. The opportunities presented by the NCETM to encourage teachers as enquirers, to provide accessible research digests, and to offer opportunity for the development of professional learning communities were all viewed as potential ways forward.

6 Policy, Curriculum and Pedagogy

Given the nature of the work of members of BSRLM, issues of curriculum and pedagogy intersect almost all areas of discussion. While we had a thematic group of 'curriculum and pedagogy' it is easy to see that many thematic groups are likely to touch on such a central element. For example, the pedagogic approaches taken by a teacher will have implications for the tools used, the extent to which mathematical thinking is prioritised or supported, the state and place of talk, and the assessment practices used. We cannot cover all of the Proceedings under each of these thematic groups here so have elected to examine the Proceedings in this area through a third theme: policy. During the period of this review, there have been substantial policy changes affecting mathematics education directly as well as education more broadly. We were interested to explore the impact of these policy directives on practice, examining how and where policy impacts on the foci of BSRLM members' work and the extent to which shifts in curriculum and pedagogy – to a greater or lesser extent influenced by policy – are evident in the Proceedings.[15]

In addition to the Proceedings *responding to* policy, BSRLM also has a role in *influencing* policy. Due to a change in the constitution occurring during the period of the review, the BSRLM Executive are able to respond to inquiries such as Williams' (2003) and McNamara & Williams' (2003) submissions to 14+ Mathematics Inquiry. These two submissions are fascinating; coming within four months of each other, they demonstrate how quickly the policy climate shifts with the second submission being deemed necessary due to the change of remit of the Smith Inquiry. It is also worth noting that part of this change of remit included a positive response from the government to a call for a network for CPD in mathematics, the need for which was emphasised in Williams' first submission. This network was the beginnings of NCETM.

Where the role of BSRLM might be strengthened is in exploring how Working Groups might both influence and respond to policy. As noted in Section 4.2.1, a number of WGs have not been sustained beyond one or two meetings and this is the case in relation to policy. There is clearly an appetite to discuss such issues with five policy related WGs formed during the period of this review. However, none were sustained – which may simply be an artefact of the closure of a consultation/enquiry period – and only one published its work:

- The Adrian Smith Enquiry (met June 2004 – unpublished)
- Mathematics Education and Policy (Lerman & Noyes, 2005)
- Communicating Research to Practitioners & Policy Makers (met November 2008 – unpublished)
- Consultation on QCA proposals for AS/A level mathematics (met June 2009 – unpublished)
- National Curriculum Review (met March 2011 – unpublished)

While Lerman & Noyes's (2005, p.109) WG met towards the beginning of our review period, the questions raised as starting points by this group still seem pertinent now. We would advise that

[15] In order to keep this task manageable, and owing to our limited awareness of policy changes in mathematics education across the world, we centre this discussion on the Proceedings considering the UK – and predominantly English – context.

these concerns are not lost or left to chance through individuals' study foci, and it may be sensible for such a WG to be reconvened:

i. How will mathematics education be shaped by the education, political and economic fields in the next 10 years?
ii. How will/might the mathematics education research community contribute to policy and/or policy thinking, whether through subject pedagogy, social and critical perspectives, curriculum theory, etc.
iii. What theoretical resources might be brought to bear on policy issues in mathematics education?
iv. What research needs doing and how can we better disseminate this to ensure high impact?

6.1 Milestones in mathematics education

As Lerman & Noyes (2005, p.110) noted, any attempt to provide a full trajectory of mathematics education policy across any time period "would of course omit important factors in the development of the subject but nonetheless this kind of work is useful in trying to theorise how we come to have the current curriculum form with its associated assessment and resource paraphernalia." We concur with this in our attempt to understand how curriculum and pedagogy have been influenced by policy as seen in the BSRLM Proceedings, and are very aware of the limitations of what we can provide here. We do not purport to have produced an exhaustive policy list (see Table 6), nor do we refer to all of the Proceedings in which the study was of, or influenced by, a particular policy directive. Instead, we provide a flavour of the enactment and impact of key policies.[16] We note that with the exception of the NNS / Primary National Strategy (PNS) there is very limited longitudinal or revisited analysis of the impact of any policy; we posit that this may be the result of such rapid policy changes and the limited possibility for any to be really embedded in practice.

Year	Key policies / reports impacting on mathematics education
1999	National Numeracy Strategy
2001	Mathematics strand of the KS3 Strategy
2004	Smith Report: Making Mathematics Count
2006	DfES Primary National Strategy (Primary Framework for mathematics)
2006	First teaching of new 2-tier GCSE Mathematics
2006	NCETM established
2008	Education and Skills Act – required education/training until 18, abolished KS3 SATs, introduced reduced KS3 curriculum
2008	Ofsted Report: Mathematics: understanding the score – noted emphasis on test preparation over mathematical understanding
2009	White Paper: Your child, your schools, our future – National Strategies abandoned
2010	Mathematics Specialist Teacher (MaST) programme
2012	Teachers' Standards with specific focus on teacher subject knowledge in primary mathematics

[16] We only consider policies related to mathematics education. Of course, curriculum and pedagogy are embedded in far-reaching wider policy directives in education such as changes to the Ofsted framework.

Year	Key policies / reports impacting on mathematics education
2013	Compulsory mathematics post-16 for those without GCSE grade C
2014	New National Curriculum (unwritten Mastery agenda takes hold)
2014	Statutory Framework for the EYFS – mathematics identified as a specific area
2014	Removal of use of calculators in any KS2 SATs tests
2014	Core Maths qualifications for 16-19 year olds with a GCSE Grade C or above
2014	First Maths Hubs established
2014	The Mathematics Teacher Exchange: China-England
2015	New mathematics GCSE specification
2015	Primary Mastery Specialist teachers programme
2016	Reasoning papers introduced into KS1 and KS2 SATs
2017	New AS/A level mathematics and Further mathematics
2019	Multiplication tables check (Year 4) – mandatory from 2020

Table 6: Key mathematics education policies and reports 2003-2017

6.2 Policy enactment

Reading across the Proceedings reveals a raft of difficulties in relation to policy enactment. At what would seem quite a fundamental level, Roper, Threlfall & Monaghan (2005) identify the difficulties inherent in terminology used within policy; in their example, the authors note that *Functional Mathematics* is a term without definition either in the academic literature or in the policy/consultation reports, comparing its vagueness with the term 'numeracy' and suggesting that different groups/individuals may see/use functional – or 'useful' – in very different ways. Bellamy (2017) also notes the 'baggage' associated with Functional Skills in her analysis of the forced GCSE resit programme; importantly this Proceeding highlights the tendency of policy reviews to focus on groups such as employers or teachers, yet silences the voices of those the policy is done to: the students. Bellamy's (2017) work also reveals a contradictory tension noted across many of the Proceedings which emphasises certain difficulties of policy change and enactment. While Bellamy highlights the negative attitudes which may arise from the forced nature of GCSE resits, Homer, Mathieson & Banner (2017) draw on literature supporting compulsory mathematics programmes post-16 in noting the lack of compulsion as a factor in the limited uptake of Core Maths. These apparently contradictory findings suggest that a 'one-size-fits-all' approach to policy may be problematic. This was certainly the deduction of Lowe & Pope (2015) who in their wide-ranging analysis of the concept of 'expected progress' from the end of KS2 to GCSE found that a common expectation could not be applied to all pupils; the expectation proved to be too low for some, yet unattainable for others.

Once enactors are brought into the picture, further tensions emerge. Golding (2012), in the context of the implementation of a new GCSE specification, illustrates how not having the sufficient conditions for change can result in varied policy enactment both between and within departments. Differences in policy enactment, referred to as 'selective implementation', were also witnessed in Venkatakrishna & Brown's (2004) exploration of the implementation of the mathematics strand of the KS3 strategy where a wide-variety of factors impacted on implementation. For some teachers, their goals and philosophies underpinned how they took on board the strategy, while for others, the – previously unseen – challenge to and stipulation of pedagogy influenced their practice. At a school level it was found that the direction of policy implementation changed; while some, as intended, used the Strategy to structure their teaching, others fitted existing schemes of work to the Strategy objectives.

6.3 'Mastering the curriculum': Pedagogy and policy in mathematics education today

The advent of the NNS brought with it something hitherto unseen in mathematics education policy: the *how* as well as the *what.* It was perhaps unsurprising, that in some cases, the policy was seen to be "constraining critical reflection on pedagogy" (Waite & Gatrell, 2005, p.178) as teachers – and particularly trainee teachers – took on the guidance without question. We might expect, in an era post-abandonment of the NNS and with a NC which only specifies the content, policy not to substantially dictate pedagogy. Indeed, it has been found, as Townsend (2015) shows, mastery is a 'nebulous' concept schools and individuals are grappling with. In the final part of this section, we examine where mastery resides in current policy and how this has been translated into pedagogy.

The subheading above is taken from Pawlik's (2016) Proceeding which offers some thought-provoking questions: is it not quite an audacious claim to say you have mastered something? Can you actually master anything? Mastery appears to be a significant 'buzzword' in mathematics education currently, yet, as Pawlik notes, 'mastery' does not appear in the NC document itself. However, the term has been pushed strongly by the NCETM and brought to educators' and researchers' attention through the Maths Hubs and Shanghai mathematics exchange project. There is a strong argument that elements pushed as aspects of 'mastery lessons' are no more than what has always been considered 'good' teaching. Indeed, the five big ideas of mastery as put forward by NCETM (coherence, representation & structure, mathematical thinking, fluency, variation, see Figure 4)[17] can all be found in the BSRLM Proceedings across the review period. Although some appear with greater intensity in the later years, they did not abruptly appear with the new focus on a discourse of mastery (see, for example, Mason's (2011) discussion of forms of variation theory, with a discussion of its history dating back to well before any mention of mastery).

Further, as Hodgen et al. (2014, p.23) observe in their discussion of different perspectives on the Shanghai mathematics exchange programme and on learning from international comparisons, learning has been bi-directional and some, "celebrated" approaches such as the bar model are "best seen as an amalgam and development of English, Dutch and Russian approaches to representation." We might even argue that learning from Shanghai in the way espoused by the teacher exchange programme is not new; as discussed in Section 5.3.3, Ding & Jones (2016) used observation and analysis of geometry teaching in Shanghai over ten years ago to propose good models of pedagogy for use in England. They concluded that "By studying lessons given by experienced mathematics teachers in China, this might inform the development of new pedagogical approaches to teaching geometry" (p.45). It does not seem too farfetched to claim that BSRLM was ahead of the game here.

[17] See: https://www.ncetm.org.uk/resources/50042

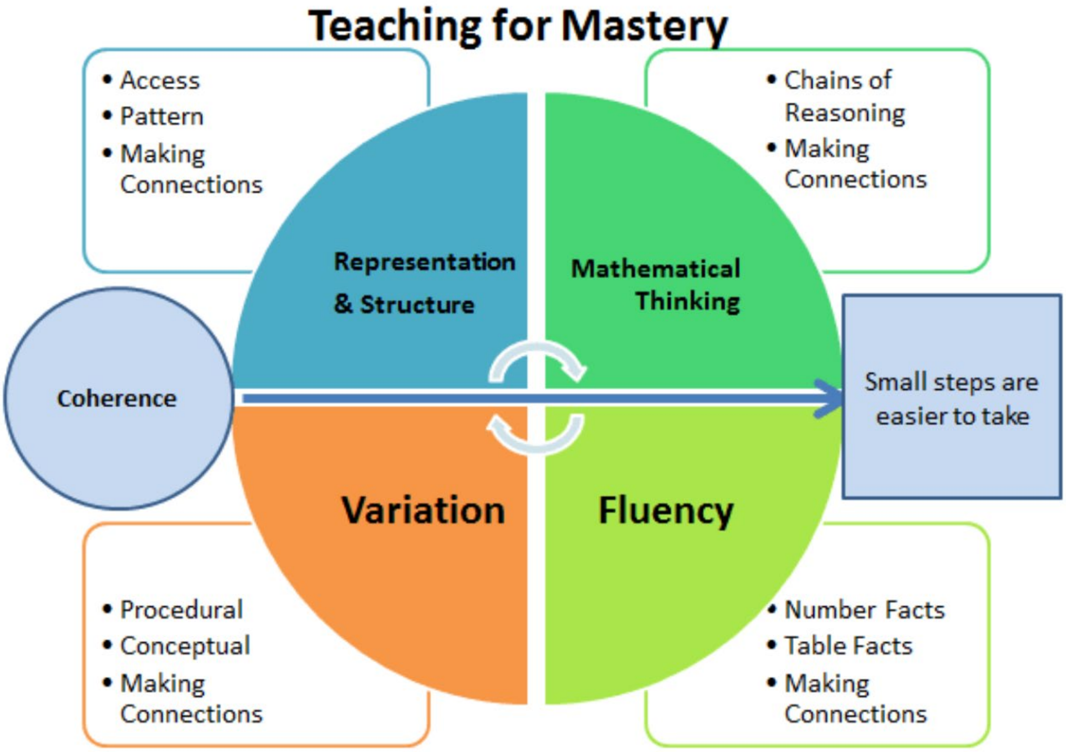


Figure 4: NCETM's five big ideas in teaching for mastery

7 Teachers and Teacher Development

Within the 773 Proceedings:

- 12% had or were assigned the keyword *Initial Teacher Training/ITT;*
- 12% had or were assigned keywords *Continuing Professional Development (CPD)* and/or *Professional Learning;*
- 11% had or were assigned keywords *Subject Knowledge, Teacher Subject Knowledge* and/or *Pedagogic Content Knowledge.*

Many authors of the Proceedings work in universities in the role of lecturers on ITT programmes with access to ITT students; this may account for the high proportion of the corpus focusing on ITT. It illustrates the important platform that BSRLM provides for sharing and discussing practice in mathematics ITT.

Teacher professional development in mathematics education is a strong theme throughout the Proceedings. Indeed, as noted in Section 5.1, CPD was one of just two main themes emerging in the Proceedings focussed on the EYFS. Joubert et al. (2008) reported preliminary results from the *Researching Effective CPD in Mathematics Education* (RECME) project. They explored teachers' explanations of why CPD was effective, and found that the word *effective* was widely interpreted by teachers and included: changes in knowledge and beliefs, organisational change, changes in practice and the value of incidental conversations with peers. One teacher described how effective CPD enabled professionals to "com[e] together to talk about real dilemmas we are faced with and to come up with ways of solving these together" (Joubert et al., 2008, p.63).

In earlier work examining the value of collaboration, Joubert (2017) analysed 49 Collaborative Teacher Projects, funded by the NCETM, in which schools worked in partnerships with expert others. Although Joubert (2017, p.6) cautioned here that it was too soon to know if the developments reported by the teachers were sustained, the collaborative projects generated "a real sense of having done something worthwhile" and illustrated the importance of teachers collaborating to design their own CPD. Later, the final RECME report, a case-study of which was reported at BSRLM (Back & Joubert, 2009a), again emphasised that collaborative groups permitted ownership by the participants of the direction of their CPD. This sustained involvement and enthusiasm over time provided a supportive arena for discussion of professional challenges. Participation such as this in collaborative networks was also the focus of Carruthers & Worthington's (2009) study, embedded in the analysis of children's mathematical graphics in the EYFS.

In addition to providing important support for professionals, collaborative CPD projects, and the strong emphasis placed by the case-study collaborative network on observation and analysis of children's mathematical activity, also shifted the focus of the group from teaching to learning, a not insignificant shift in thinking. Back & Joubert (2009b) considered that this close consideration of the processes of young children's learning was significant in the impact of this network on the development of participants' professional practice.

7.1 Teachers as researchers

Teacher research as an approach to CPD initiatives is a recurrent theme within the Proceedings. The themes of collaboration, teachers designing their own CPD and a focus on pupil learning are evident in the Proceedings addressing teacher development. These are similarly evident in those focusing on teachers as researchers.

The NCETM funded teacher development projects between 2006 and 2011 (NCETM, 2018). Joubert & Sutherland's (2010) analysis of the projects arising from this initiative propose that the writing of an end of project report may have facilitated teachers' reflection. They argue that many reports produced by NCETM grant holders make a "significant and worthwhile contribution to mathematics education in England and wider afield" (2010, p.35).

However, they raise concerns that this work is not sufficiently recognised within the mathematics education community. There is some evidence to support this concern; within the Proceedings we reviewed, less than 4% were authored (singularly or in conjunction with HEI partners) by practitioners and report on practitioner enquiry. It could be argued that this is not a key focus of BSRLM, but it may have implications for how the outcomes of other funded teacher small-scale enquiry projects are reported and built on in the mathematics education community.

Lesson study is an example of research informed teacher enquiry. Lesson study first emerged as an approach to CPD in England in 2001 (Dudley, 2012) involving the application of cycles of collaborative planning, teaching, observation and evaluation, with the aim of improving pupil learning. The Proceedings may reflect a developing interest in Lesson Study, for example, Corcoran et al. (2011); Deshler (2015); Omuvwie (2015) and Ramirez (2017).

Archer (2016) examined Lesson Study as an approach to development in ITT provision. In this small-scale study, she argues that the knowledge constructed through focused reflection afforded by lesson study is empowering, and, moreover, that this could enable teachers who trained using lesson study to be better placed to challenge accepted practice in schools. However, applying a lesson study approach is not without complexity. Rempe-Gillen's (2013) case-study in English schools explored an approach to CPD that comprised joint planning, lesson observation and post-lesson discussion. She reported instances in which teachers elected not to engage in these collaborative aspects of the CPD because of the association of these practices with novice teachers. Consequently, she raised concerns that the culture in England might not be conducive to the successful use of Lesson Study.

7.2 Teacher subject knowledge: BSRLM as a developmental sphere

At the beginning of the 21st century in England, and prior to the period of this review, there were significant policy changes relating to trainee teachers' mathematics subject knowledge. This policy climate, discussed briefly below, perhaps stimulated a focus on teacher subject knowledge with more than 10% of the Proceedings addressing this focus, including 7 of the 20 Proceedings in November 2003 specifically focusing on teacher subject knowledge.

The 1998 DfE Ciruclar 98/4 announced policy that, in England, the ITT NC for Primary/Secondary Mathematics needed to be taught to all ITT primary and secondary mathematics students respectively. This included:

- the pedagogic and subject knowledge & understanding that students were required to demonstrate;
- an expectation that HEIs would audit students' subject knowledge against both the ITT and pupils' national curricula and address identified gaps.

A subsequent addendum in 2000 detailed the introduction of compulsory mathematics skills tests for all ITT students in England. These policy changes were concurrent with the introduction, in 1999, of the NNS in England. The 2003 Ofsted report of the Implementation of the National Literacy and Numeracy Strategies reported that weak teacher subject knowledge was a consistent characteristic of unsatisfactory teaching, reinforcing the need to address teachers' mathematical subject knowledge.

The Proceedings relating to subject knowledge encompass a broad range of approaches and foci. For example, Hodgen (2003) synthesised findings presented at the *Recent UK Research into Prospective Primary Teachers' Subject Knowledge colloquium.* He posed important questions to the BSRLM and mathematics education community about the depth of required mathematics subject knowledge, trainees' confidence and anxiety in mathematics and the value of subject knowledge audits. Murphy's (2003) study used questionnaires to enquire into trainee teachers' views of mathematics subject knowledge audits. Others examined practitioner subject knowledge in relation to specific mathematical topics, e.g. Mooney, Fletcher & Jones' (2003) study relating to geometry and probability.

Throughout the review period, BSRLM has provided an important forum for discussing and sharing research throughout developmental phases. One example of this was the development of the Subject Knowledge Quartet, first presented in 2003. Huckstep, Rowland & Thwaites' (2003) videotape study of mathematics lessons used a grounded approach, setting out a tentative subject knowledge framework comprising four categories: theoretical backgrounds and belief; transformation, presentation and explanation; coherence and contingent action. They acknowledged the emergent nature of the findings and their intention to further develop the framework: "At present our groupings are tentative and our conceptualisation of the four categories is subject to ongoing deliberation and modification" (Huckstep, Rowland & Thwaites, 2003, p.39). Nine months later, Rowland, Huckstep & Thwaites (2003) presented *The Knowledge Quartet*, in which the four dimensions were refined as: foundation, transformation, connection and contingency (see Figure 5). These were illustrated through a case-lesson of a teacher in her first year of practice.

Two years later, the team returned to present a new layer to the Subject Knowledge Quartet, in which the framework was used in video stimulated recall interviews to support a trainee teacher to reflect on her mathematics teaching (Thwaites, Huckstep & Rowland, 2005). This work developed the potential of the framework to research teachers' mathematical subject knowledge; it introduced the idea that the framework could also be utilised by mentors to engage trainee teachers in reflective dialogue to support the development of subject knowledge.

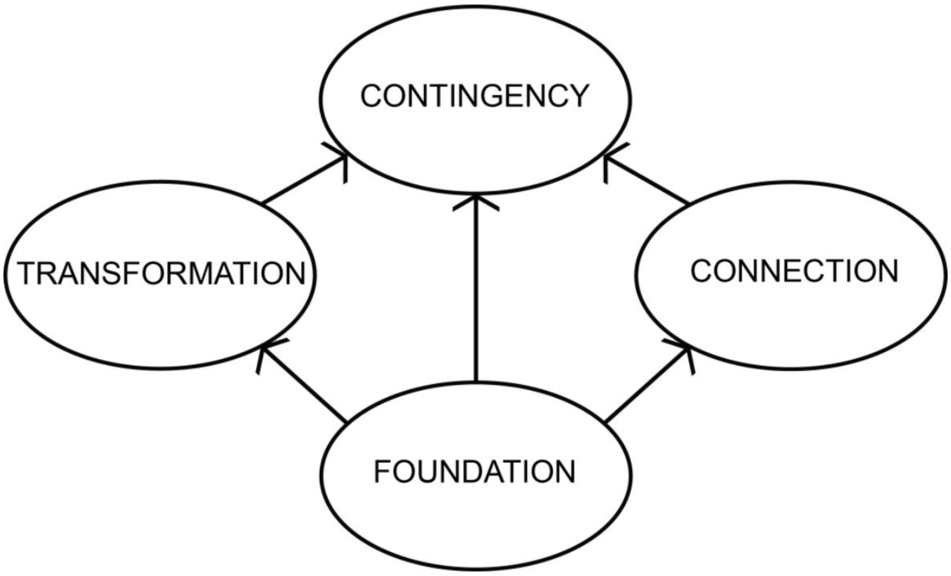


Figure 5: The four dimensions of the Knowledge Quartet (Rowland & Weston, 2012)

A characteristic of research in development being discussed and disseminated at BSRLM is the space developed for others to build on, contribute to and apply the ideas in their own studies, something which was evident in the case of the Knowledge Quartet:

- In a study on the characteristics of lesson observation feedback given to trainees, Harris (2006) used the contingency dimension of the KQ to code feedback relating to trainees' deviation from their agenda, response to children's ideas and use of opportunities.
- Kleve's (2009) Norwegian study of a primary (elementary) teacher's knowledge of fractions also used the KQ as an analytic framework to identify and understand contingent moments.
- Huntley (2011a, 2011b) used the KQ in his analytical framework of his doctoral study into primary trainees' choice of examples. He researched trainees on an undergraduate teacher training route with a grade C in GCSE mathematic, and found that their choices were not pedagogically planned.

A common theme across the original Knowledge Quartet studies, and subsequent related research, is the application of the framework to the practice of individuals and the resulting fine-grained data. One outcome of this approach is the development of a coding manual for the Knowledge Quartet (Weston, Kleve & Rowland, 2012), intended to be applicable in primary and secondary settings to support data analysis and professional reflection in relation to teachers' mathematical subject knowledge.

Rowland and colleagues' work, presented and discussed in part through BSRLM Conferences and Proceedings, led to a range of outputs in books, journals and online (e.g. Rowland, Huckstep, & Thwaites, 2005; Rowland & Turner, 2009; Rowland & Weston, 2012; Rowland, 2014), showcasing the role of BSRLM as a sphere for the development of robust, longitudinal work, as well as broadening the reach of such work to a wider audience. Of course, this is not the only example of a project being developed across multiple years at BSRLM (see for example the discussion of ICCAMS in Section 5.3.3) but it aptly demonstrates the role the Society can play in supporting progression from the development of early ideas to the widespread uptake of resultant outcomes.

8 Summary & Recommendations

In this section we bring together our findings from across our analysis. We note the many and significant strengths of the corpus, evidence of an active, engaged, engaging and developing society. Many of the concerns raised in Nickson's (2003) earlier review have been addressed, fully or partly. We see evidence of a growing membership, not only in terms of actual numbers, but importantly in terms of reach, with more teacher-practitioners, international researchers, and novel partnerships emerging. There is of course still room for development in all these areas and we note this, in addition to other specific gaps, in Section 8.2. In concluding the review, we make a series of recommendations to BSRLM, its Executive and membership for potential future research directions and developments for the Society.

8.1 Strengths of the Proceedings (2003-2017)

The strengths of the corpus are many and varied, an artefact of the vast range of topics and populations covered, research approaches employed, and the inclusive nature of the Society. The themes which emerged in this analysis are strongly aligned with those coming through in international reviews, suggesting the work conducted within BSRLM to be representative of, and hence able to speak to and with, the wider field of mathematics education.

The published Proceedings show support for research – and researchers – at all stages of development, from pilot studies to funded large-scale/longitudinal projects and from students and ECRs to established researchers and research teams. Teachers and practitioner-researchers are beginning to be brought into the fold – although there is work to be done here – with the BSRLM Blog established during the period of the review evidence of a concern for engaging a wider audience.

A major shift in emphasis from Nickson's (2003) review to this present review is in the age-phases covered. The call for a greater emphasis on the primary phase has clearly been heeded, with over a quarter of studies now focussed on this phase. These Proceedings, with a focus on the primary phase, covered our full range of thematic groups. They explore issues pertinent across mathematics education as well as those specific to the phase, either by virtue of pupils' age-related development or through an exploration of the impact of policies directed at the primary phase, of which there have been several.

A second major change has been in the intense focus on teachers and particularly on teacher development, hence us covering this specifically in Section 7. We now have a significant body of work on teacher subject knowledge and CPD. We also know the importance of teacher involvement in – or teacher-designed – CPD. Further, collaboration and the ability to engage in supportive networks seem central to productive professional development.

8.2 Gaps in coverage

While work on CPD has developed, it is still the case that the vast majority of the Proceedings are authored by researchers affiliated to HEIs (see Section 4.1). Very few teachers are presenting their research independently at BSRLM. While there is a debate to be had as to the purpose of BSRLM, in an era where teachers are being encouraged – and in some cases funded – to engage in small-scale teacher enquiry, there is a need to consider whether BSRLM has a role to play in supporting dissemination of these and in potentially developing teacher-researcher networks.

We noted above that the stronger focus on studies looking at the primary phase was to be commended. However, this disguises an important issue. Although Primary, Secondary (including KS5) and Higher Education all receive significant attention, other phases, notably FE and those at the age extremes – Early Years and Adult Education – are almost absent. Each of these neglected groups represent sizable proportions of learners and important aspects of the overall picture in understanding learning and teaching in mathematics education. The formation of the Early Years Working Group and a recent policy focus on EYFS may go some way to addressing one aspect of the current imbalance. Further, although we found that 15% of the Proceedings looked across phases, it was interesting to note that, unlike the rest of the corpus, very few of these were empirical. This suggests a current gap in studies looking empirically at teaching and learning across age-phases, which may account for a concern raised in a number of the Proceedings about the lack of focus on learning trajectories and, related to this, SEND, where learners are not following the expected developmental trajectory.

A final concern in relation to gaps relates to the imbalance in author location. Given that BSRLM represents UK mathematics Education, we are not concerned to find that 83% of the authors of the Proceedings are affiliated to UK institutions. We are concerned, however, that over 60% of these are affiliated to institutions in the south of England. The five Day Conferences generating the highest number of published Proceedings were all in the South of England: UCL IoE (London), King's College London, Oxford (twice) and Cambridge. We question whether this has implications for the generalisability of findings across the UK and suggest it is prudent that further work is done to explore the implications of this imbalance, and begin to redress this through consciously promoting BSRLM to under-represented areas of the UK.

8.3 Recommendations

We finish this review with seven specific recommendations in relation to the focus and dissemination of BSRLM research, and support for all members' work:

i. The population gaps identified above – EYFS, FE, adult-education, and SEND – could be developed (as is the case of EYFS) through the instigation of Working Groups or future Day Conferences with a 'special focus' or research colloquia.

ii. The very small proportion of the Proceedings addressing the development of mathematics understanding in the EYFS gives rise to a recommendation to develop closer links with EYFS professional networks, drawing on the positives identified in CPD studies.

iii. There were clear signs of methodological innovation in the Proceedings, for example in the use of video analysis. We recommend that the Society supports members to further such methodological innovation in the future, and in particular examine how they might encourage the scaling up of small-scale empirical research.

iv. The Executive may wish to discuss the changing membership, the focus of the Society and its role in supporting practitioner-researchers and in disseminating research to a teacher audience.

v. There are clearly examples of the potential strength of the Working Groups. Ways might be established to support the longevity and membership of these groups. One area to explore is the use of these groups to track and respond to the many policy changes impacting on mathematics education in the UK.

vi. Wider readership/citation could be supported through a facility to search the keywords of the Proceedings on the BSRLM website.

vii. New Researcher Days and workshop sessions such as Merrilyn Goos' *Writing for Publication in Mathematics Education Research Journals: Issues, Challenges, and Strategies* (June 2018) provide support for new and established writers; further similar sessions should be considered and the resultant advice disseminated, perhaps via the BSRLM Blog. Future writing workshops might include:

- Writing an abstract and selecting keywords
- Disseminating data analysis methods
- Writing Proceedings papers (perhaps with a 'to include' checklist, but avoiding being over-prescriptive)
- Developing Proceedings into articles for RME

We have thoroughly enjoyed surveying the Proceedings and writing this review; we look forward to the future development of the Society and the opportunity to witness future developments, perhaps addressing the issues raised in this review.

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