Provoking mathematical awareness: supporting lower attaining primary school pupils to make meaningful contribution in mixed attainment pairs

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This UK classroom based doctoral research employed a pedagogical focus on mathematical awareness. Three primary class teachers worked alongside the researcher to design lessons which aimed to provoke pupil awareness of the mathematical properties and structures embedded in their mathematical activities. Through this focus we sought to enable lower attaining primary school pupils to make meaningful mathematical contributions to the progress of tasks in the context of mixed attainment pair working. Video recording captured the activity and interaction of lower attaining pupils and their partners. Analysis focused on the nature of the mathematical awarenesses demonstrated by the lower attaining pupil and the impact of these on mathematical progress for the pupil pair.

Each of the lower attaining pupils developed and demonstrated important mathematical awarenesses demonstrating the potential of lower attainers to make valid contributions to mixed pair working.

Key words: mathematical awareness; lower attainers; mixed attainment; primary

Mathematical awareness

Gattegno argues that knowledge "comes to life through awareness" (1987, p51). His positioning of awareness at the centre of the learning process leads Young and Messum to assert that awareness can be considered to be the "conceptual unit of learning" (2011, p83). In working with pupil awareness in the context of the current study, I build on Mason's (2008) interpretation of awareness as a sensitivity in both detecting and responding to stimuli. My definition of mathematical awareness focuses on sensitivity to stimuli: *the detection of, or recognition of the applicability of, mathematical features, relations, or processes*.

Awareness is closely linked to the constructs of attention and noticing. Here, Hewitt (2001) suggests a reciprocal relationship between awareness and attention, with the focus of attention influencing that of which we become aware, and awareness guiding the focusing of attention. Mason (2008) further links noticing and attention by identifying noticing as a shift in one's attention. Whilst at times noticing may be spontaneous, Mason (2011) argues that an intentional shift of attention is frequently required for one to become consciously aware of something. In the current study, the term 'noticing' was felt to be more accessible to pupils than the term 'awareness' and was thus used in classrooms throughout.

In relation to mathematics learning, the related constructs of awareness and noticing have been productively harnessed, albeit in a small number of studies, in both evaluating pupil learning and in tracking its development. For example, Mulligan and Mitchelmore (2009) evaluated pupil responses to a range of mathematical tasks in terms of the awareness of mathematical structure that the responses demonstrated. Their finding that higher degrees of structural awareness correlated with higher mathematical achievement underpins their recommendation for a greater focus on developing pupils' awareness of mathematical structure. Separately, and focusing on observation of pupils' mathematical activity, Lobato, Hohensee and Rhodehamel (2013) conclude that what pupils notice mathematically has important consequences for their subsequent reasoning. Significant also is that pupil noticing appears to have ramifications beyond the individual lesson activity (Hohensee, 2016; Lobato et al., 2013). Specifically, what one notices and uses in one situation can inform both the way one looks and what one looks for in a new situation. Thus, it may be that encouragement to notice mathematical relationships may inform pupils' independent noticing and use of such relationships in novel situations. These findings provide support for Hewitt's (2001) call for a focus on awareness in an attempt to awaken the mathematician inside the pupil, supporting independent decision making and action. Indeed, Gattegno's prime concern of the education of awareness (1987; 2010), that is to say the focus on the nature of an awareness rather than on any specific outcome of it, also appears to be supported by such findings.

Lower attainers in mixed attainment groups

Research relating to the use of mixed attainment groupings, and to the impact of these groupings for lower attainers, is limited. Where it exists, this research does not always address primary aged pupils and is not always classroom based. However, available evidence indicates that mixed group structures benefit the lower attainer. Exposure to a wider range of mathematical topics and access to the thinking and reasoning of higher attainers are identified as instrumental in supporting mathematical progress for lower attaining pupils (Burris, Heubert, & Levin, 2006; Fawcett & Garton, 2005). These studies also suggest that learning gains for the higher attainer in such mixed groups derive from the requirement to explain their thinking and reasoning.

Notable in reviewing the above studies is that gains for both higher and lower attainer alike are seen as deriving from the contributions of the higher attainer; what the lower attainer can contribute mathematically to such mixed attainment working is not established. This study seeks to contribute understanding of how the recognised potential of lower attainers to think and operate mathematically when working with teachers or other lower attainers (Dickinson, Eade, Gough, & Hough, 2010; Houssart, 2004) can be productively harnessed to enable lower attainers to make valid mathematical contributions in the context of a mixed attainment pair.

Indeed, evidence from studies focusing on promoting awareness suggest that this pedagogical focus may be particularly supportive for lower attainers. Hohensee (2016) suggests that a focus on noticing had particular impact on pupils who were underachieving at the outset of his study. His results indicated that these pupils made greater gains in their awareness and use of the mathematical relations on which his study was based. This leads Hohensee (2016, p89) to hypothesise that noticing 'may be a mechanism that could be leveraged to help students who are low achieving to catch up with students who are achieving at more proficient levels'. Voutsina and Ismail (2011) also argue for the benefit of a focus on awareness for low attainers. They suggest that their focus on awareness altered the goal of the task from solving to explaining and that this was important in increasing lower attaining pupils' sensitivity to the additive relations that were the focus of the study.

Working with pupil awareness

Attempts to identify what pupils notice, attend to, or are aware of, are complicated by the partially internal nature of these constructs; awareness is an event that is beyond the reach of the observer. In view of this, my analytic focus is on pupils' responses to the stimuli that they detect. Thus, I begin analysis by noting development or change in the observable action and interaction and use this to infer the awarenesses that underpin the changes observed.

However, whilst awareness guides and informs actions and decisions, attempts to infer or intuit awarenesses from observed actions or from speech must be approached with caution (Hewitt 2001; Mason 2008). Actions may, for example, suggest understanding of a mathematical idea when only competence in a procedure is secure; speech may incorporate the language of mathematical generality when the pupil is referring to an individual case only (Mason 2008). This notwithstanding, Hewitt (2001) argues that teachers can, with care, draw on their own awarenesses in inferring pupils' awarenesses in their actions and speech.

The study

The study took place in three primary classrooms (one year 4 class and two year 6 classes) over the course of one academic year. One preliminary lesson established a baseline in terms of pupil awareness and the operation of mixed attainment pairs. Subsequently three research lessons utilising open structured tasks which had strong affordances for awareness and for the construction of reasoning were jointly planned between the researcher and the three class teachers; class teachers led all lessons. In addition to the affordances of the task, the pedagogical focus on mathematical awareness comprised the following components: promotion of the enactive use of representations to represent numbers and task situations; teacher focus on 'what do you notice?' through class introduction, displayed speech bubbles and interventions at pupils' tables; and recording sheets which asked for recording of noticing.

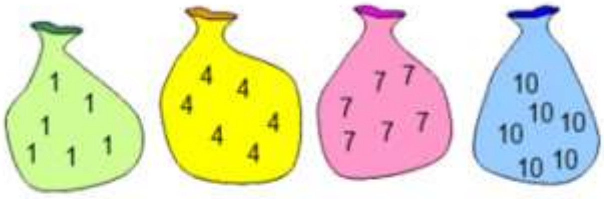
Two focus lower attainers, with no identified special educational need and working with the age related curriculum but functioning below age related expectations, were identified in each classroom. Their action and interaction, as they worked with a partner functioning at above age related expectations, was video recorded. Subsequent data analysis focused on four of these six lower attainers.

I identified four categories of mathematical awareness relevant to the tasks in which pupils were engaged: awareness of mathematical processes; awareness of mathematical patterns and properties; awareness of structure and generality; awareness of the rules of the activity. Of these four, the second and third (awareness of patterns and property and awareness of structure and generality) were the prime foci for analysis. Analysis of pupil awareness of mathematical processes supported explanation of the development of awarenesses in the second and third category. Pupil awareness of the rules of the activity was used to identify the starting point for focused analysis for each pupil pair.

Findings

Across all research lessons, the pedagogical focus on mathematical awareness was associated with lower attaining pupils' expressed structural awarenesses making a significant contribution to task progress for the pupil pair. There was a more limited impact on lower attaining pupils' awareness of mathematical patterns and properties.

Here I present findings in relation to two lower attainers' structural awarenesses from one of the tasks utilised. The task, 'What numbers can we make?' was drawn from nrich (n.d.) and presented the challenge below:



'These bags contain an inexhaustible supply of 1s, 4s, 7s and 10s. Choose any three numbers from any combination of the bags and add them together. Try this a few times. What do you notice?'

http://nrich.maths.org/7405

Pupils made several random selections of numbers and relatively swiftly, established that the total of the three numbers was always a multiple of 3. Pupils were then asked to try to explain this finding and were encouraged to make use of Numicon in representing the numbers involved. Significant in this lesson is that three of the four lower attainers contributed a significant structural awareness that enabled reasoning about the task to progress. In each of these cases, it was evident from the higher attainer's response that they had not developed the awareness at the time that it was expressed by the lower attainer

The following dialogue extract from Mia (lower attainer) and Ruby (higher attainer) illustrates both the awareness developed and the higher attainer's response:

2:1 Mia takes a sharp in breath, smiles and says 'ah'. She points at the Numicon: 'in all three numbers, oh, um, they have like, one left over'

2:8 Mia picks up the Numicon 4, putting it on the table in front of her and places a 3 on top of it. 'If you like take 3 away from that you have one left' (see fig 1)

2:10 Mia: 'you get the same with this one' she puts two threes on the 7(see fig 1); Ruby leans back, smiles and releases a long sigh. Mia continues: 'and you would get the same with 10 as well' she puts three 3s on the 10 showing the one left over

2:11 The Numicon is oriented so that the remainders are close together. Mia places a further Numicon three in the space created; 'it makes another three and it ends up being in the three times table' (see fig 2)

Fig 1: Mia places threes on top of the '4' and '7' Numicon

Fig 2: Mia orients the Numicon to add the final three piece

Mia demonstrates a structural awareness in turns 2:1, 2:8 and 2:10 that each of the numbers can be considered as a multiple of 3 plus 1. This represents a shift from her earlier contemplation of each number as a single object (a 'seven', a 'ten'). Ruby's smile and sigh (2:8) indicates that she has now developed the same awareness. Mia's modelling leaves the 'one left over' uncovered by Numicon and this supports her later reasoning. With some prompting from the class teacher the 4, 7 and 10 pieces were oriented (as shown in fig 2) so that the uncovered 'remainders' were adjacent to each other. Her positioning of the additional 3 piece (shown in her hands in fig 2) in the space created, provides a strong representation of her awareness that when three numbers that are all one more than a multiple of three are added together the ‘remainder’ from each one makes an additional 3. Her articulation in turn 2:11 is confirmation of her awareness of this relationship. This structural awareness moved the pair on from an impasse and enabled them to satisfactorily complete the task presented. It also supported predictions in an extension activity.

In a parallel lesson, lower attainer Rose, working with higher attainer Hannah, made a different contribution. Hannah articulated the relationship to multiples of 3 for each of the numbers 4,7, and 10, but was unable to explain this relationship for the fourth number in the set: 1.

3:6 Hannah: ’yeah cos four is, one less is three and seven, one less is six’. Picking up a Numicon 1 piece and holding it up, she says in impassioned tones: ‘And the ‘one’ I have no idea about’

3:7 Rose takes the ‘one’ piece from Hannah and is about to speak as Hannah exclaims: ‘the one is just, a one!’

3:8 Rose jumps in quickly: ‘no, no, no, if it was one less it would be zero and there would be nothing there. And that would be a multiple of 3. Zero, three, six..’

3:9 Hannah, excitedly and with mouth open wide: ‘oh yeah!.. Oh yeah!’ Rose smiles very broadly.

In turn 3:8 Rose shows that she can follow and extend the pattern of considering one less than the number in the set. Moreover she can use the same form of articulation as her partner in justifying her thinking. Her utterance in 3:8 evidences her awareness that 1 has an equivalent relationship to multiples of 3 as do the other numbers in the set (i.e. that all are one more than a multiple of 3). The significance of this awareness lies in its impact on the task for Rose and her partner. The identification of shared commonality across all four of the numbers supported the generality that ensued and the girls’ confidence in it. Significantly also, the response from both girls to this awareness; excitement from Hannah and delight from Rose, served to give the task further momentum and raised Rose’s status in the task.

Conclusions

Data from all research lessons reveals that mathematical awarenesses demonstrated by lower attainers directly contributed to task progress for both pupils in the pair. Thus, I argue that in mixed attainment pairs, gains for the lower attainer do not arise purely through their exposure to the higher quality thinking and interaction of their partners. This is not to deny the importance of such exposure for the lower attainer that these partnerships can afford, indeed many such examples exist in the data. However, lower attainers in mixed attainment partnerships also gained through the development of their own mathematical awarenesses; as illustrated above, higher attainers similarly gained from the awarenesses that their lower attaining partner contributed. Rather than supporting Hohensee’s (2016) suggestion that a focus on noticing supports lower attainers to catch up with their peers, I suggest that the focus on awareness in this study has served to awaken the mathematician(Hewitt, 2001) in the lower attainer. With the focus shifted from solution finding to noticing and explaining (Voutsina & Ismail, 2011), lower attainers have demonstrated that they are able to contribute meaningfully to the mathematical progress of a mixed attainment pair. Moreover, the potential of such positive mathematical experiences to strengthen the confidence of lower attainers in the value of their own mathematical contributions indicates that a focus on awareness has much to offer.

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