Non-linear and mixed regression models in predicting sustainable concrete strength

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Abstract

Most previous research adopting the regression analysis to capture the relationship between concrete properties and mixture-design-related variables was based on the linear approach. This had limited accuracy. This study applies non-linear and mixed regression analyses to model properties of environmentally friendly concrete based on a comprehensive set of variables containing alternative or waste materials. It was found that best-fit non-linear and mixed models achieved similar accuracies and superior R^2 values compared to the linear approach, with both the numerical and relative input methods. Individual materials' effects on concrete strength were statistically quantified at different curing ages using the best-fit models.

Keywords

Sustainable concrete; concrete mixture design; cementitious materials; concrete strength; statistical modeling; non-linear regression analysis; mixed model

1. Introduction

As the most widely consumed construction material worldwide, concrete has recently caught much attention from researchers who are interested in discovering how its sustainability could be improved by replacing conventional cementitious and aggregate materials with alternative or waste materials. In their studies [1, 2, 3], one major focus is to understand how these materials affect concrete properties. Although some understanding has been generated based on limited experimental data, it is not adequate to stimulate wider application of sustainable concrete in the construction industry. To fill this gap, the authors selected Portland limestone cement (PLC), Haydite® lightweight aggregate (LWA), and fly ash (FA) Class F as alternative materials in their sustainable concrete research. These selections were based on the industry feedback collected from a market survey that had been conducted earlier [4].

Mathematical modeling has been adopted by some researchers [1, 5, 6, 7] to capture the relationships between properties of sustainable concrete and mixture-design-based independent variables. So far, limited studies such as Omran et al. [7] have included a comprehensive list of concrete-mixture-design-based inputs (especially the different replacement rates of alternative or waste materials) in the quantitative methods to predict concrete properties. Also, the independent variables from concrete mixture design can be numerically-based (e.g., Omran et al. [7] and Chithra et al. [8]) or relative-based (e.g., Topçu and Saridemir [9]; Omran et al. [10]). There is, however, limited research that compares the prediction performance between these two input systems.

Previous research (e.g., Atici [6]; Chithra et al. [8]) found that the regression analysis was reliable in predicting concrete strength, but less accurate than the Artificial Neural Network (ANN). However, the multivariate regression analysis approach has its advantages, since it does not require programming or additional time for model training. This enables it to generate easy-to-use regression constants, and estimate the significance of input variables. So far, the regression analysis has not been thoroughly explored in concrete mixture design, particularly in the use of non-linear or mixed models, which could be a better-fit [11].

The contributions of this research lie in: 1) proposing and testing non-linear and mixed regression models as an alternative approach to the traditionally linear method in predicting concrete strength; 2) adopting a complete set of mixture-design-related variables for modeling environmentally friendly concrete; 3) comparing the prediction performance by using the numerical and relative input methods in a comprehensive set of statistical models; and 4) providing a statistical guide for studying the effects of alternative/waste materials on concrete strength at different curing ages. This research also compares the performance of non-linear and mixed methods with existing approaches, such as ANN and other data mining methods.

The remainder of this paper is organized as follows. Section 2 provides the background information on the study area. Section 3 describes the materials used in the experiment, the experimental design, and the proposed statistical models. Section 4 presents the prediction performance of the tested models and other statistical analysis results. Section 5 discusses the robustness of non-linear and mixed models in predicting sustainable concrete strength, and Section 6 concludes this paper.

2. Background

2.1. Concrete mixture design

Concrete contains four basic ingredients: cement, water, fine aggregate, and coarse aggregate. Chemical admixtures such as air-entraining admixture (AEA) could also be added into concrete to achieve varied properties. In the concrete industry, guidelines are usually used for designing concrete mixtures. The overdesign factor is statistically determined by experimental data, or is calculated based on specific formulas when no sufficient data is available [12]. Internationally, the concrete mixture design approach can be divided into two major systems: numerical and relative systems. Examples of numerically-featured mixture designs include the Absolute Volume Method, introduced in ACI 211 [13], and the Design of Normal Concrete Mixes described in Building Research Establishment [14]. The relative system-based mixture design includes the Equal Paste Volume method [15], which considers the mix proportions of the concrete. These proportions include the water–binder ratio, paste to aggregate ratio, and sand to coarse aggregate ratio.

The early market survey [4] of U.S. concrete suppliers and prefabricators nationwide confirmed that most of industry practitioners (81% of total 39 survey respondents) used industry guidelines and standards for concrete mixture design. In addition, respondents also mentioned using companies' historical data (used by 68% of respondents) and other methods such as trial batches (used by 19% of respondents) in their mixture design. Among various methods applied by the industry, there are limited applications of quantitative methods (e.g., statistical tools) in modeling or predicting how the mixture design affects concrete properties. However, according to some existing studies [6, 16, 17], there is great potential for quantitative methods to be used in the concrete mixture design. This will benefit concrete companies that may have limited budgets, but need to investigate mix proportions to obtain the desired concrete strength [6, 18]. In addition, the compressive strength (CS) of concrete during the early curing age is usually unknown, but this information is of great importance to the structure to be built, as well as in site operations [6].

2.2. Sustainable concrete

Producing conventional concrete utilizes large amounts of natural resources (e.g., sand and rock) while generating significant energy and environmental impacts from the manufacturing of Portland cement (PC) [4]. Environmentally friendly or sustainable concrete refers to concrete with lowered life cycle environmental impacts, which is accomplished by replacing conventional ingredients with recycled waste materials, locally available materials, or alternative materials associated with lower greenhouse gas emissions or improved concrete properties (e.g., durability). In the U.S. concrete industry, the top three most commonly used supplementary cementitious materials (SCMs) had been identified as fly ash (FA), silica fume, and ground-granulated blast-furnace slag (BFS) based on the market surveys of both Jin et al. [4] and Obla [19]. The applied alternative aggregates were limited to LWA and recycled concrete aggregate (RCA) [4]. Although various other waste or alternative concrete materials (e.g., Berry et al. [20], Binici [21], Topçu and Boğa [1]) had been studied, their industry application is limited for various reasons, such as limited material sources or regional availability.

Many researchers have performed experimental tests to study the effects of waste or alternative materials on concrete properties, such as the studies of oyster shell [22], RCA [23], and the research on FA Class C and furnace slag [24]. In concrete research, simple linear plots were commonly used (e.g., Basri et al. [25]; Berry et al. [20]; Bondar et al. [26]) to relate the concrete properties (e.g., CS) to a given independent variable (e.g., age). Although there were some limited studies attempting to link concrete properties to multiple independent variables in concrete mixture design with various substitution rates of waste or alternative materials, it was not sufficiently quantified how the different substitution rates of such materials impact concrete properties. In addition, the study of concrete properties in relation to an alternative material usually requires a large amount of experimental data, which is not only time-consuming, but also cost-prohibitive. Therefore, most previous studies on environmentally friendly concrete rarely investigated more than one alternative concrete material (e.g., Bondar [26]; Topçu and Boğa [1]; Yang et al., [22]).

2.3. Prediction methods linking concrete mixture design to strength

Applying statistical and mathematical models in the research of cement/concrete-related construction materials is not new. Aderibigbe et al. [16] described the relationship between compressive strength and optimum water to cement ratios (*w/c*) using a power curve equation for cement/clay soil mixed blocks. Similarly, other studies, for example, Topçu and Saridemir [9], adopted statistical analyses to describe the relationships between concrete properties (i.e., strength) and aggregate proportion by using a linear regression equation. In these studies, only one variable was considered, e.g., *w/c* or percentage of supplementary aggregate. Nevertheless, concrete mixture design involves multiple interrelated factors (e.g., *w/c* and substitution rate of SCMs). It would be necessary to study how the concrete properties can be affected by the presence of these factors, i.e., joint effects from the mixture design.

Table 1 provides details from a few representative studies that have adopted the linear regression approach to model the relationship between concrete strength and mixture-design-related variables. It can be seen that the regression models adopted in these studies had relatively low determination coefficients (i.e., R^2 value). This level of accuracy appears to be lower than that achieved by some machine learning techniques [6, 7]. As pointed out by St-Pierre [11], the traditional simple regression methods are likely to generate biased statistical results and the mixed model methodologies could be applied to provide more accurate predictions. So far, the non-linear and mixed models have been tried in fields such as biological engineering [11, 27], but their application in the concrete-materials-related studies is still sparse.

Table 1
Existing regression models used to predict concrete strength.

Reference	Independent variables	Adopted model	Achieved R^2
Yeh [28]	Cement, FA, BFS, water, superplasticizer, coarse and fine aggregates, and curing age	Linear regression	0.574
Deepa et al. [29]	Cement, BFS, FA, water, superplasticizer, coarse and fine aggregates, and curing age	Linear regression	0.491
Atici [6]	Proportion of BFS, FA, curing age, rebound number	Multiple linear regression	0.899

		analysis (MRA)	
Chou et al. [30]	Cement, FA, BFS, water, superplasticizer, coarse and fine aggregate, and curing age	MRA	0.611
Chithra et al. [8]	Cement, fine and coarse aggregate, silica, slag, superplasticizer	MRA	0.672

In the literature review, ANN was found to be the most widely used modeling approach in predicting concrete properties [18, 28, 31, 32, 33, 34, 35, 36]. ANN can automatically build the relationships between inputs and outputs through a learning algorithm. However, using this approach depends on software applications and requires larger and more varied training dataset(s) [6]. A few concrete property studies [37, 38, 39] used fuzzy logic (FL). This method mimics human thinking to deal with problems caused by the imprecision of source(s) in consideration of linguistic uncertainties [5, 38]. The use of ANN in predicting concrete properties could be complicated by the large number of variables [31]. The same problem also applies to FL due to its characteristics of human-like manner and linguistic rules [38]. Also, statistical models can address the inverse problem within concrete mixture design, while ANN faces difficulties in solving such problems [6].

3. Materials and Methods

3.1. Materials used

In this study, PC Type I/II with 28-day CS at 38 MPa, brown sand (fine aggregate), and pea gravel with maximum size at 9.5 mm (coarse aggregate) were selected as conventional concrete materials used in the control group of the experimental tests. While PC Type I/II (for general use) and brown sand are widely used in concrete production, pea gravel is more frequently adopted in lab-based experimental studies, as opposed to crushed stones. These materials are locally available in many parts of the world. Unconventional concrete materials used in this study include PLC Type GUL (General Use Limestone Cement), FA Class F, and Haydite LWA (Size B, similar to the size of pea gravel). Jin et al. [4] and Omran et al. [7] defined PLC, FA, and LWA were defined as alternative or waste materials that improve concrete sustainability or environmental friendliness, as they would either reduce the cement carbon footprint, save materials, or achieve other environmental benefits. In this study, Micro Air was chosen as the AEA to increase the air content in the concrete batches.

Suppliers provided the Mill Test Reports for PC Type I/II, PLC (GUL), and FA Class F. Table 2 lists the major elements of these materials. Other minor ingredients such as K_2O in FA and C_3A in PC are not listed. PLC can be produced by intergrinding or blending PC with limestone, which reduces the carbon footprint of cement manufacturing. The PLC used in this study was interground with 12% limestone, as calculated based on its CO_2 content, which is defined in ASTM C150 Standard Specification for Portland Cement [40].

Table 2
Mill Test Reports of cementitious materials in this study (percentage by weight).

Cementitious material	SiO_2 (%)	Al_2O_3 (%)	Fe_2O_3 (%)	CaO (%)	MgO (%)	SO_3 (%)	Alkalis (%)	Loss on ignition (%)	Autoclave expansion (%)
PC	20.1	5.0	3.3	63.2	2.4	2.6	0.56	2.0	0.02
PLC	18.4	4.6	3.0	59.9	2.9	3.6	0.65	5.2	0.08
FA	43.7	21.0	23.8	5.0	1.0	1.7	1.97	1.5	0.00

The chemical analysis of oven dry Haydite is shown in Table 3. The density information of the three aggregate materials is listed in Table 4. While the loose bulk dry density information was provided by the suppliers, the oven dry density and specific gravity were obtained following the standard test methods described in ASTM C127 for coarse aggregate [41] and C128 for fine aggregate [42].

Table 3
Chemical analysis of Haydite (provided by the supplier).

Item	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	SO_3	Na_2O	K_2O	TiO_2	P_2O_5	Mn_2O_3	SrO	Cr_2O_3	ZnO
Weight (%)	60.36	19.95	8.09	2.41	2.40	0.13	0.92	4.58	0.96	0.11	0.15	0.01	0.03	0.04

Table 4
Dry densities of aggregates used in this study.

Type of aggregate	Loose bulk dry density[a] (kg/m^3)	Oven dry density[b] (kg/m^3)	Specific gravity[c]	Fineness modulus
Pea gravel	1602	2643	2.64	6.01
Haydite Size B	673	1298	1.30	5.39
Brown sand	1602	2611	2.61	2.48

[a]Loose bulk dry density is the mass of dry aggregate per unit volume of aggregate particles, including the volume of impermeable pores and water-filled voids within the particles, and the pores between the particles.
[b]Oven dry density is defined by ASTM C127 and C128 as the mass of oven dry aggregate per unit volume of aggregate, including the volume of impermeable pores and water-filled voids within the particles but excluding pores between particles.
[c]Specific gravity (or relative density), according to ASTM C127 and C128, is the ratio of the oven dry density of the material to the density of distilled water (assuming 1000 kg/m^3).

3.2. Experimental design

The mixture design, which incorporates different proportions of waste or alternative materials, is displayed in Fig. 1.

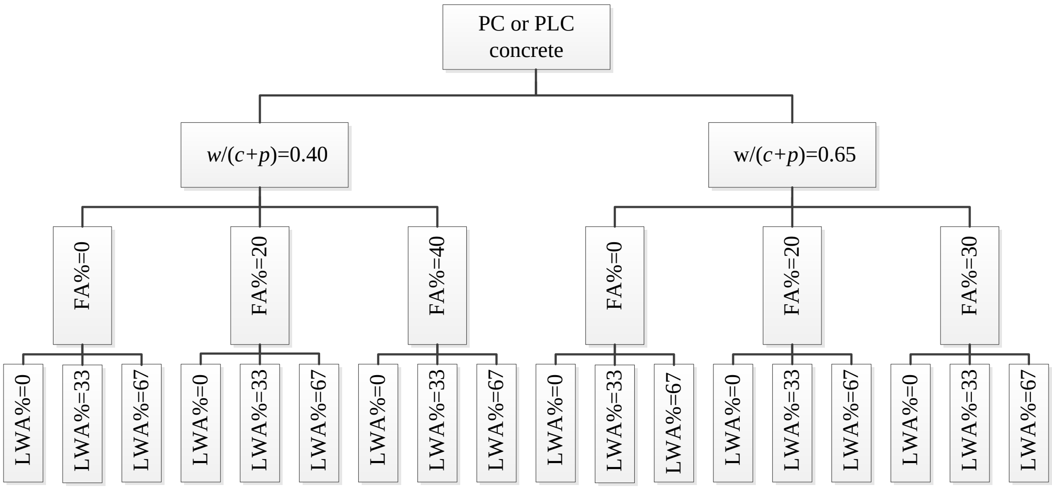

Fig. 1. Mixture design (36 batches).

The 36 batches represent combinations of different cement types (i.e., PC or PLC), *w/(c+p)* (water to cementitious material weight) ratios, substitution rates of FA Class F (FA%) by weight of the cementitious material, and replacement rates of Haydite to pea gravel by volume. This study implemented both a lower *w/(c+p)* ratio (0.40) and a higher *w/(c+p)* ratio (0.65), which are typical used in concrete mixture design to meet different quality requirements (e.g., strength and durability). Having at least two different ratios was also necessary for the statistical analysis as *w/(c+p)* is one of the key independent variables in the relative system. The making, pouring, and curing of the concrete followed the guidelines of ASTM C31/C31M-06 [43]. Strength tests were based on 10 mm by 20 mm cylinders cast in single-use plastic molds, which were cured at room temperature (23 °C), and tested at different ages: 3, 7, 28 and 90 days. Compressive strength (CS) and split tensile strength (TS) tests followed ASTM C39/C39–05 [44] and ASTM C 496/C496M-11 [45], respectively. The mixture design details can be found in Table 5.

Table 5
Design of concrete mixture proportions.

Mixture batch				Ingredients per cubic meter of concrete						
Cement type	*w/(c+p)* ratio	FA (%)	Haydite (%)	Water (kg)	Cement (kg)	FA (kg)	Sand (kg)	Pea gravel (kg)	Haydite (kg)	Micro air (ml)
PC	0.4	0	0	211	528	0	742	750	0	135
			33	211	528	0	742	504	121	135
			67	211	528	0	742	247	247	135
		20	0	211	422	106	742	750	0	135
			33	211	422	106	742	504	121	135
			67	211	422	106	742	247	247	135
		40	0	211	317	211	742	750	0	135
			33	211	317	211	742	504	121	135
			67	211	317	211	742	247	247	135
	0.65	0	0	211	324	0	902	750	0	112
			33	211	324	0	902	504	121	112
			67	211	324	0	902	247	247	112
		20	0	211	259	65	742	750	0	112
			33	211	259	65	742	504	121	112
			67	211	259	65	742	247	247	112
		30	0	211	227	97	742	750	0	112
			33	211	227	97	742	504	121	112
			67	211	227	97	742	247	247	112
PLC	0.4	0	0	211	528	0	742	750	0	135
			33	211	528	0	742	504	121	135
			67	211	528	0	742	247	247	135
		20	0	211	422	106	742	750	0	135
			33	211	422	106	742	504	121	135
			67	211	422	106	742	247	247	135
		40	0	211	317	211	742	750	0	135
			33	211	317	211	742	504	121	135
			67	211	317	211	742	247	247	135
	0.65	0	0	211	324	0	902	750	0	112
			33	211	324	0	902	504	121	112
			67	211	324	0	902	247	247	112
		20	0	211	259	65	742	750	0	112
			33	211	259	65	742	504	121	112
			67	211	259	65	742	247	247	112
		30	0	211	227	97	742	750	0	112
			33	211	227	97	742	504	121	112
			67	211	227	97	742	247	247	112

3.3 Non-linear and mixed regression models in predicting concrete strength

This study aimed to explore the potential relationship between sustainable concrete strength and input variables (i.e., concrete mixture-based variables and curing age) by applying statistical models. Besides the conventional linear regression model, introduced as Model 1 in Eq. (1), this research proposed alternative non-linear and mixed models to improve the determination coefficient when predicting concrete strength based on the mixture-design-related variables. These models range from Model 2 to Model ($2k$ + 3) in Eqs. (2)-(5), where k denotes the number of independent predictor variables (IPVs) in the regression model (it is 9 and 8 for the numerical and relative input methods, respectively). The equations for all of these models are displayed below:

Model 1: Multi-linear regression analysis

$$Y = \alpha + \sum_1^k \beta_i X_i, \ i = 1,2 \ldots \ldots k \qquad (1)$$

Model 2: A non-linear model involving natural logarithms

$$\ln Y = \alpha + \sum_1^k \beta_i X_i, \qquad i = 1, 2, \ldots . k \qquad (2)$$

Model 3: A second type of non-linear model involving natural logarithms

$$\ln Y = \alpha + \sum_1^k \beta_i \ln X_i, \qquad i = 1, 2, \ldots . k \qquad (3)$$

Mixed models from (4) to (k+3)

$$\frac{X_i}{Y} = \alpha + \sum_1^k \beta_i X_i, \ i = 1,2 \ldots . k \qquad (4)$$

k mixed models with natural logarithm

$$\frac{\ln X_i}{Y} = \alpha + \sum_1^k \beta_i \ln X_i, i = 1,2 \ldots k \qquad (5)$$

In these models, X_i represents *k* IPVs such as curing age, *Y* is the response random variable (RRV) referring to CS or TS, and α and β denote constants. Only Model 1 from the above (2*k*+3) models is linear, and all the remaining non-linear or mixed relationships were converted into linear formats. The statistics software, Minitab, was used to analyze these (2*k*+3) models. The values of R^2 and residual standard deviation were generated to compare the accuracy of these models in predicting each target RRV. The *F* and *p* values generated from an Analysis of Variance (ANOVA) were used to test the significance of the selected regression model (at a 95% level of significance) in describing the data samples. The null hypothesis is that the target RRV cannot be predicted using the selected model with the chosen IPVs. A *p* value less than 0.05 from ANOVA would reject the null hypothesis and indicate that the selected regression model fits the data. Residual analysis was also conducted in Minitab to study the distribution and values of residuals, which were the differences between the predicted RRV and experimental data. The Durbin-Watson statistical test is based on the null hypothesis that residuals from a least-square regression are not autocorrelated [46]. The Durbin-Watson value ranges from 0 to 4, and a value near 2 indicates non-autocorrelation. The ideal Durbin-Watson value would fall between 1.5 and 2.5 [6, 8].

Among the *k* IPVs, some may have more significant effects on the target *RRV* than others. The *t*-test of correlation analysis was used to determine the significance regarding the effect of each IPV on RRV. There is a *p* value corresponding to each *t* value for an IPV. At the 95% confidence level, a *p* value lower than 0.05 would indicate that this selected IPV makes a significant contribution to RRV. In contrast, IPVs with *p* values higher than 0.05 are those without significant contributions. A possible reason that some IPVs had higher significance than others is the strong internal correlation among IPVs, which caused redundancies. Therefore, the regression analysis could be redone by removing the redundant IPVs, shortening the equation to include only significant IPVs. Target RRVs (Y_1 and Y_2) and various IPVs using both numerical and relative input systems are defined in Table 6.

Table 6
Definitions of RRVs and IPVs in the numeric and relative systems.

	Definitions	
Variables	Numeric system	Relative system
Y_1	Concrete CS (MPa)	Concrete CS (MPa)
Y_2	Concrete TS (MPa)	Concrete TS (MPa)
X_1	Concrete age (days)	Concrete age (days)
X_2	W (kg): Amount of water used in the mixture of per m^3 of concrete	w/(c+p): Water-cementitious material ratio
X_3	PC (kg): Amount of PC used in the mixture of per m^3 of concrete	PLC%: Replacement of PLC to PC*
X_4	PLC (kg): Amount of PLC used in the mixture of per m^3 of concrete	FA%: FA substitution rate in cementitious material
X_5	FA (kg): Amount of FA used in the mixture of per m^3 of concrete	LWA%: Haydite LWA substitution rate in coarse aggregate
X_6	S (kg): Amount of sand used in the mixture of per m^3 of concrete	S/(c+p): Weight ratio of sand to cementitious material
X_7	CA (kg): Amount of coarse aggregate used in the mixture of per m^3 of concrete	S/CA: Volume ratio of sand to coarse aggregate
X_8	LWA (kg): Amount of Haydite used in the mixture of per m^3 of concrete	Unit AEA (ml): Amount of air entrainment (ml) per 100 kg of cement (AEA)
X_9	AEA (ml): Amount of air entrainment used in the mixture of per m^3 of concrete	N.A.

*: X_3 in the relative system is a binary value, with its value at 0 when using PC and 1 when PLC is used.

4. Results

In this study, the two major input systems within concrete mixture design (i.e., numerical and relative input systems) were compared for their accuracy in predicting concrete strength. The best-fit models were identified under each input system. By removing significantly correlated IPVs within each input system, the regression modeling process was rerun by shortlisting. Finally, the whole data sample was divided by the different curing ages to study the effects of each IPV on concrete strength at various ages.

4.1. Comparison between the numerical and relative input systems

The regression analysis for both CS and TS was conducted based on the trial of 21 and 19 proposed models for numerical and relative input systems, respectively. The reliability of these models was compared, and the best-fit model was identified for each of the four scenarios, i.e., concrete CS and TS in these two input systems. Table 7 displays the corresponding R^2 values for all CS and TS predictions using both systems.

Table 7
Statistical modeling results in the numerical and relative systems.

Statistical approach	Model no.	Predication of CS				Predication of TS			
		Numerical system		Relative system		Numerical system		Relative system	
		RRV	R^2	RRV	R^2	RRV	R^2	RRV	R^2
Linear	1	CS	0.907	CS	0.901	TS	0.764	TS	0.775
Non-linear	2	ln(CS)	0.876	ln(CS)	0.878	ln(TS)	0.732	ln(TS)	0.748
	3	ln(CS)	0.953*	ln(CS)	0.934*	ln(TS)	0.866	ln(TS)	0.836
Mixed models	4	Age/CS	0.932	Age/CS	0.933	Age/TS	0.952*	Age/TS	0.955*
	5	W/CS	0.740	(w/(c+p))/CS	0.807	W/TS	0.626	(w/(c+p))/TS	0.774
	6	PC/CS	0.823	PLC%/CS	0.823	PC/TS	0.899	PLC%/TS	0.859
	7	PLC/CS	0.813	FA%/CS	0.832	PLC/TS	0.878	FA%/TS	0.873
	8	FA/CS	0.839	LWA%/CS	0.816	FA/TS	0.890	LWA%/TS	0.868
	9	S/CS	0.788	(S/(c+p))/CS	0.830	S/TS	0.726	(S/(c+p))/TS	0.814
	10	CA/CS	0.822	(S/CA)/CS	0.793	CA/TS	0.818	(S/CA)/TS	0.736
	11	LWA/CS	0.874	Unit AEA/CS	0.772	LWA/TS	0.874	Unit AEA/TS	0.694
	12	AEA/CS	0.698	ln(Age)/CS	0.906	AEA/TS	0.632	ln(Age)/TS	0.884
	13	ln(Age)/CS	0.914	ln(w/(c+p))/CS	0.839	ln(Age)/TS	0.900	ln(w/(c+p))/TS	0.804
	14	ln(W)/CS	0.859	ln(PLC%)/CS	0.841	ln(W)/TS	0.798	ln(PLC%)/TS	0.902
	15	ln(PC)/CS	0.838	ln(FA%)/CS	0.822	ln(PC)/TS	0.904	ln(FA%)/TS	0.898
	16	ln(PLC)/CS	0.837	ln(LWA%)/CS	0.862	ln(PLC)/TS	0.901	ln(LWA%)/TS	0.890
	17	ln(FA)/CS	0.862	ln(S/(c+p))/CS	0.879	ln(FA)/TS	0.911	ln(S/(c+p))/TS	0.878
	18	ln(S)/CS	0.861	ln(S/CA)/CS	0.884	ln(S)/TS	0.804	ln(S/CA)/TS	0.898
	19	ln(CA)/CS	0.881	ln(Unit AEA)/CS	0.846	ln(CA)/TS	0.881	ln(Unit AEA)/TS	0.771
	20	ln(LWA)/CS	0.841	N/A	N/A	ln(LWA)/TS	0.895	N/A	N/A
	21	ln(AEA)/CS	0.857	N/A	N/A	ln(AEA)/TS	0.782	N/A	N/A

*Model that achieves the highest R^2 value for the given scenario.

As shown in Table 7, both numerical and relative input systems led to highly consistent R^2 values from Models 1 to 4 for predicting CS, meaning similar prediction accuracy. Model 4 (the mixed model using *Age/Strength* as the RRV) achieved consistently high R^2 values for all four scenarios. All of the corresponding Durbin-Watson values in the 16 scenarios are within the reasonable range (i.e., 1.5 to 2.5). Model 4 also achieved the highest R^2 value for the predication of TS in both systems. In the CS-related RRV regression analysis, Model 3 (the non-linear approach) represents the best-fit model by achieving even higher accuracy than Model 4, the highest based on both input systems. The remaining mixed models had relatively lower R^2 values for both input systems. The R^2 values resulting from the best-fit non-linear and mixed regression models in this research (ranging from 0.934 to 0.955) are significantly higher than the values generated from previous studies adopting linear methods. This can be seen in Table 1. The accuracy level of these regression models is also comparable to that achieved by data mining techniques in Omran et al. [7] when the same dataset for CS was used.

4.2. Regression analysis using the best-fit models

Although both numerical and relative input systems had highly consistent R^2 values for the best-fit models, the former is deemed more practical for field applications due to the wide adoption of the numerically-featured ACI method of mix design [13] in North America and many parts of the world. Due to space limitations, this section only showcases the best-fit models for predicting CS and TS based on the numerical input system. However, the modeling process and outcomes of the best-fit models based on the relative input system are expected to be similar.

Compared to the R^2 values (0.907 and 0.763 for CS and TS, respectively) associated with the linear approach (Model 1), the best-fit non-linear (i.e., Model 3) and mixed (i.e. Model 4) models performed the best. Model 3, in the regression analysis for CS, provided the highest correlation, with an R^2 value of 0.953 (followed by Model 4 with R^2 value at 0.932). Model 4 achieved the highest accuracy, with an R^2 value of 0.952 for predicting TS. The two equations generated from Models 3 and 4 are listed below:

For predicting CS

$$\mathrm{Ln}(Y_1) = 6.520 + 0.212\ln X_1 - 0.056\ln X_2 + 0.808\ln X_3 + 0.817\ln X_4 + 0.006\ln X_5 - 0.775\ln X_6 + 0.014\ln X_7 - 0.009\ln X_8 + 0.177\ln X_9 \quad (6)$$

For predicting TS

$$X_1 / Y_2 = -12.500 + 0.252X_1 + 0.012X_2 - 0.003X_3 - 0.008X_4 - 0.001X_5 + 0.010X_6 + 0.007X_7 + 0.016X_8 - 0.005X_9 \quad (7)$$

Fig. 2 shows the comparison between the predicted RRVs and the experimental results. The R^2 values over 0.950 in Figs. 2(c) and 2(d) indicate the high accuracy of the identified best-fit models (i.e., Model 3 for CS-related RRVs, and Model 4 for TS-related RRVs) in predicting concrete strength-related RRVs. Model 4, which sets *Age/TS* as the RRV, tends to be non-continuous compared to Model 3 due to the large variation in curing age (i.e., Day 3, 7, 28 and 90) involved in the RRV. The discontinuous nature of RRV in the mixed model would also affect the residual distribution. As a comparison, the R^2 performance of Model 1, the linear regression approach, is also displayed in Figs. 2(a) and 2(b). It can be observed that, compared to the linear approach, non-linear and mixed methods improved the prediction accuracy of concrete strength-based RRVs.

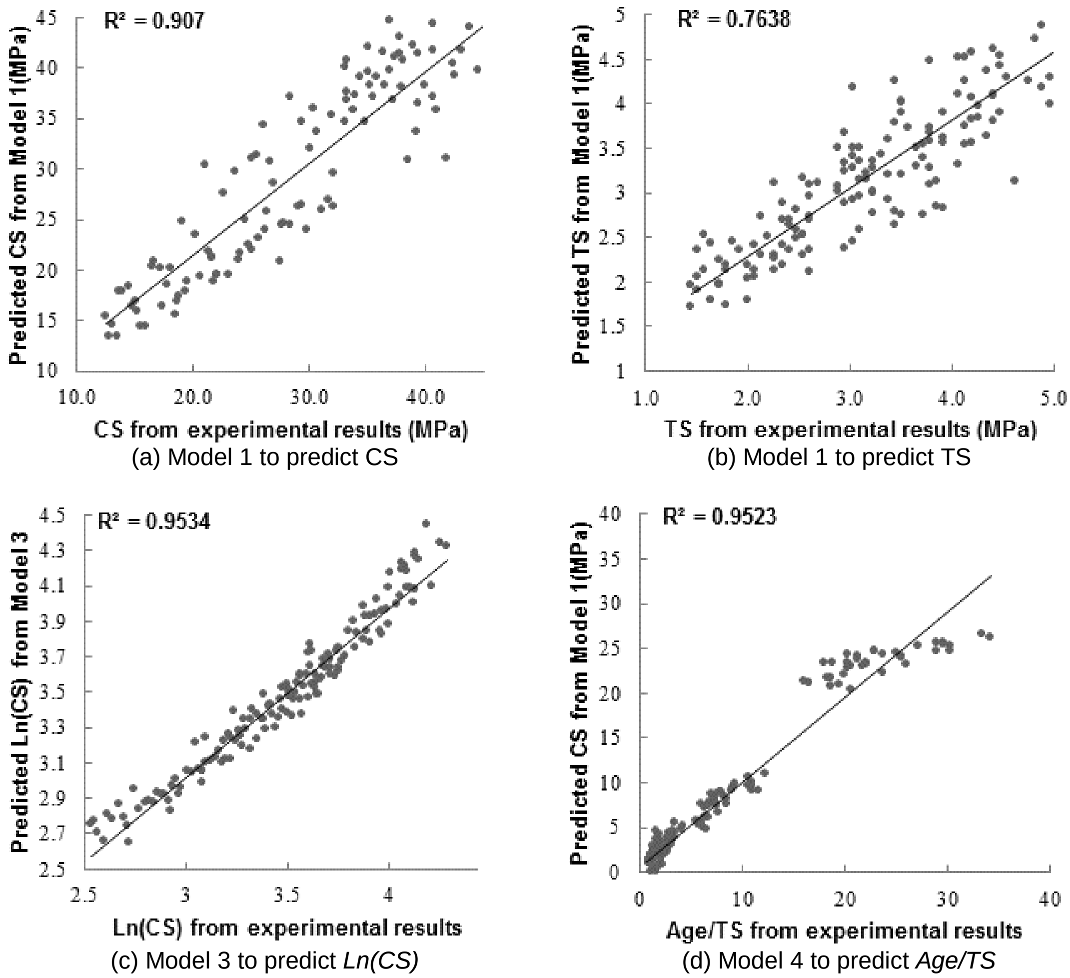

Fig. 2. Comparison between predicted RRV and experimental data using linear regression analysis and best-fit models.

A residual analysis for the best-fit models was conducted using Minitab. Fig. 3 illustrates the residual analysis results for *ln*(CS) from Model 3. The residual values of Model 3 applied in *ln*(CS) analysis presented satisfactory trends of normal distribution. This is shown in both the normal probability plot and in the histogram. The residual values appeared symmetrically distributed along the neutral horizontal line (when the residual is 0) and were not affected by the increase of fitted values. The observation order in Fig. 3 corresponds to the growth of concrete age; there were 36 observations for each of the four concrete ages (i.e., Day 3, 7, 28, and 90). Generally, the residual was not affected by curing age. Similar distribution of residual values in Model 3 could be found when applied in the relative system.

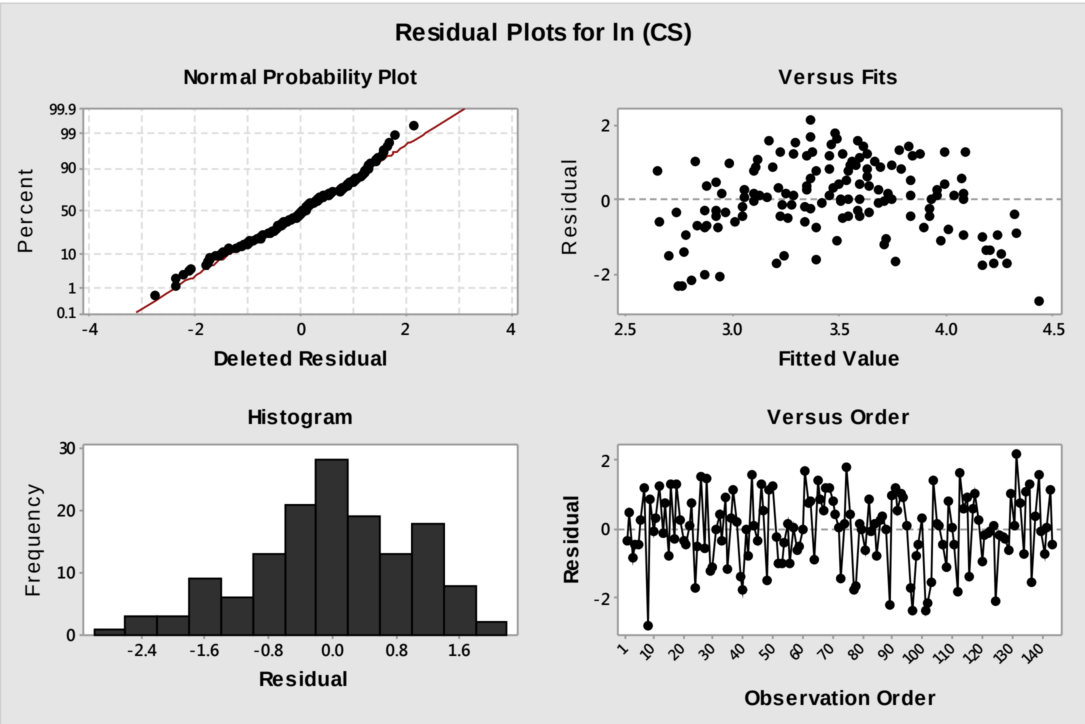

Fig. 3. Residual analysis of Model 3 in predicting *ln*(CS).

For Model 4 applied in TS, the residual distribution displayed in Fig. 4 shows less symmetry along the neutral line. Corresponding to the larger variation nature of *Age* involved in the RRV, the residual value in Model 4 tends to grow alongside the RRV value.

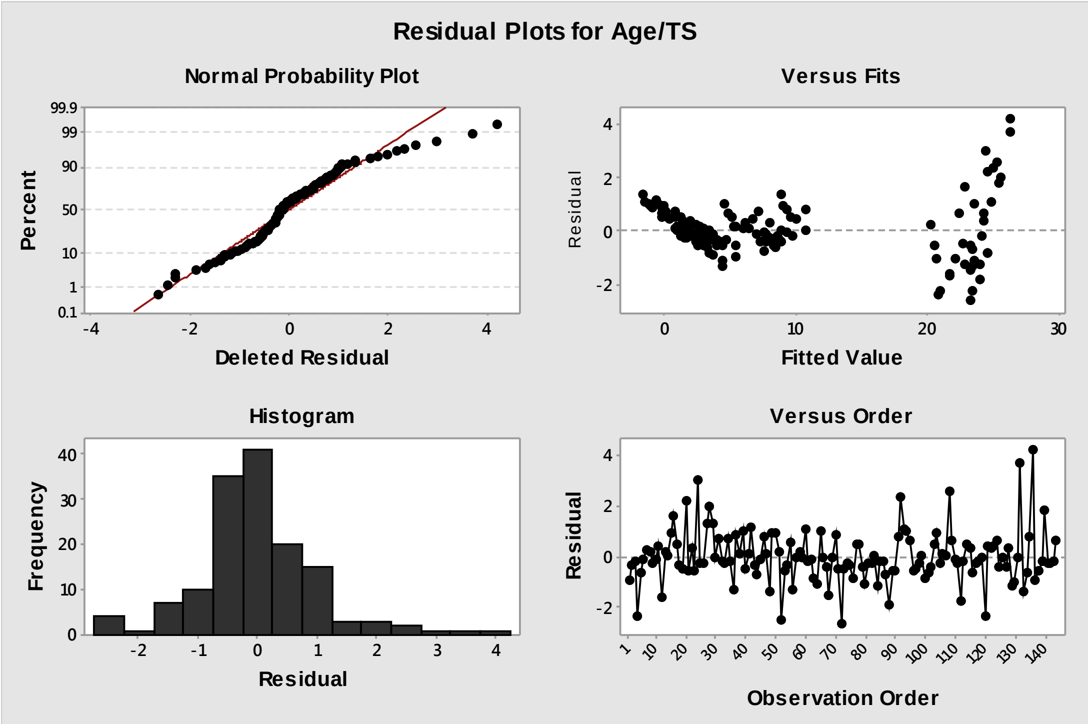

Fig. 4. Residual analysis of Model 4 in predicting *Age/TS.*

4.3. Internal correlation analysis of IPVs based on the best-fit model

This section uses Model 3 in CS to demonstrate the internal correlation analysis of IPVs and regression analysis with shortened IPVs. Pearson correlations and corresponding *p* values displayed in Table 8 indicate the correlations among mixture-design-related IPVs. Curing age was found to be independent of any other mixture-based IPVs, and sand amounts had significantly negative correlations with the CA amount. Therefore, only one IPV between sand and CA amounts needs to be kept for the shortened input variables. This study purposely kept IPVs related to the studied alternative or waste materials to capture their effects on concrete properties, which fits the research goals.

Table 8.
Pearson correlations among nine IPVs.

		1	2	3	4	5	6	7	8	9
1. ln(Age)	Correlation	1.000								
	p value	0.000								
2. ln(W)	Correlation	0.000	1.000							
	p value	1.000	0.000							
3. ln(PC)	Correlation	0.000	-0.207	1.000						
	p value	1.000	0.013*	0.000						
4. ln(PLC)	Correlation	0.000	0.237	-0.999	1.000					
	p value	1.000	0.004*	0.000*	0.000					
5. ln(FA)	Correlation	0.000	-0.375	-0.013	-0.014	1.000				
	p value	1.000	0.000*	0.876	0.867	0.000				
6. ln(S)	Correlation	0.000	-0.597	0.033	-0.071	0.011	1.000			
	p value	1.000	0.000*	0.876	0.395	0.894	0.000			

7. ln(CA)	Correlation	0.000	-0.086	-0.010	0.012	0.321	-0.177	1.000		
	p value	1.000	0.307	0.905	0.891	0.000*	0.034*	0.000		
8. ln(LWA)	Correlation	0.000	0.081	0.007	-0.009	-0.008	-0.036	-0.347	1.000	
	p value	1.000	0.333	0.938	0.914	0.927	0.666	0.000*	0.000	
9. ln(AEA)	Correlation	0.000	0.522	-0.085	0.120	-0.028	-0.855	0.208	-0.103	1.000
	p value	1.000	0.000*	0.313	0.152	0.742	0.000*	0.012*	0.221	0.000

*Significant correlations between two IPVs with *p* values less than 0.05.

Table 9 displays the regression analysis results of Model 3 for both nine IPVs and shortened IPVs. In the secondary run of Model 3, all the five of the shortlisted IPVs showed significant influences on RRV (i.e., ln(CS)). Age had the most significant impact, according to its corresponding *t* value (24.28). The negative coefficient values corresponding to FA, sand, and LWA indicate that these three materials would generally reduce concrete CS. In contrast, PLC is indicated to increase concrete CS based on the positive coefficient value and low *p* value at 0.001. It is also worth noting that the shortlisted IPVs in the secondary run of Model 3 resulted in only slightly lower R^2 at 0.907 and slightly higher residual standard deviation. However, the Durbin-Watson value fell out of the ideal range between 1.5 and 2.5. In comparison, the mixed model (i.e., Model 4) turns out to have a superior Durbin-Watson value when only the same five shortlisted input variables are retained.

Table 9.
Non-linear regression analysis results from Model 3.

		Coefficient analysis			Residual Standard Deviation		ANOVA		Durbin-Watson value
Response	Predictor	Coefficient	*t* value	*p* value		R^2	*F* value	*p* value	
ln(CS)	Constant	6.520	3.13	0.002	0.098	0.953	304.69	0.000	1.906
	ln(Age)	0.212	33.94	0.000					
	ln(W)	-0.056	-0.43	0.669*					
	ln(PC)	0.808	9.00	0.000					
	ln(PLC)	0.817	9.07	0.000					
	ln(FA)	0.006	1.68	0.096*					
	ln(S)	-0.775	-3.37	0.001					
	ln(CA)	0.014	3.69	0.000					
	ln(LWA)	-0.009	-4.56	0.000					
	ln(AEA)	0.177	3.11	0.002					
ln(CS)	Constant	21.890	29.14	0.000	0.136	0.906	266.86	0.000	1.405
	ln(Age)	0.212	24.28	0.000					
	ln(PLC)	0.007	3.27	0.001					
	ln(FA)	-0.016	-6.47	0.000					
	ln(S)	-2.819	-25.23	0.000					
	ln(LWA)	-0.017	-6.92	0.000					

**p* value higher than 0.05 indicating less significant of the target predictor on concrete-strength-based response.

4.4. Subsamples at different curing ages

Continuing the work in Jin [47], where experimental observations were obtained on the waste or alternative materials' effects on concrete properties at different curing ages, this study provided the statistical approach to test these observations. Based on the shortened IPV list from Section 4.3, the 144 total observations were divided into subsamples according to the curing age (i.e., Day 3, 7, 28, and 90). This was done to analyze the effects of multiple alternative or waste materials on concrete strength as it ages. Table 10 displays the data analysis results by rerunning Model 3 as an example.

Table 10.
Non-linear regression analysis results from Model 3.

		Coefficient analysis			Residual		ANOVA		Durbin-
Response	Predictor	Coefficient	*t* value	*p* value	Standard Deviation	R^2	*F* value	*p* value	Watson value
ln(CS) in Day 3	ln(PLC)	0.006	1.02	0.314*	0.170	0.843	41.5	0.000	1.784
	ln(FA)	-0.019	-2.96	0.006					
	ln(S)	-3.362	-12.08	0.000					
	ln(LWA)	-0.019	-3.09	0.004					
ln(CS) in Day 7	ln(PLC)	0.009	2.12	0.042	0.134	0.873	53.21	0.000	1.723
	ln(FA)	-0.022	-4.27	0.000					
	ln(S)	-2.871	-13.06	0.000					
	ln(LWA)	-0.020	-4.09	0.000					
ln(CS) in Day 28	ln(PLC)	0.009	2.61	0.014	0.110	0.895	65.8	0.000	1.625
	ln(FA)	-0.020	-4.88	0.000					
	ln(S)	-2.640	-14.65	0.000					
	ln(LWA)	-0.014	-3.60	0.001					
ln(CS) in Day 90	ln(PLC)	0.005	1.37	0.179*	0.170	0.873	53.33	0.000	2.056
	ln(FA)	-0.006	-1.44	0.160*					
	ln(S)	-2.402	-13.89	0.000					
	ln(LWA)	-0.015	-3.97	0.000					

*p value higher than 0.05 indicating less significant of the target predictor on concrete-strength-based response.

The coefficient analysis in Table 10 conveys the information that the three adopted alternative or waste materials (i.e., PLC, FA, and LWA) tended to have significant effects on concrete strength at different curing ages, with a few exceptions. Overall PLC increased concrete CS while FA and LWA decreased CS. Consistent R^2 and ANOVA analysis results were also found in Model 3 when applied to the four different concrete ages. The Durbin-Watson values all fell into the ideal range. However, compared to early ages, the effects of FA and PLC on Day 90 concrete tended to be less significant with corresponding p values higher than 0.05. This would indicate that FA and PLC tended to affect concrete strength more heavily in earlier ages (i.e., Day 7 and Day 28). However, the long-term strength of sustainable concrete would be more comparable to that of conventional concrete. This statistical finding was consistent with and supported by earlier studies [47] when comparing the concrete strength between sustainable concrete and conventional concrete using bar chart illustrations. The TS-related numerical or relative system also led to consistent findings.

5. Discussion

Although only Model 3's statistical performance was demonstrated in this paper in detail, Model 4, when applied in either TS-related numerical or relative system, was also found to have consistent results following the procedures described in Sections 4.3 and 4.4. This suggests the robustness of non-linear and mixed models in predicting concrete mechanical properties based on both numerical and relative systems. Although non-linear models might not have ideal Durbin-Watson values when IPV is shortlisted, and mixed models might not have ideal distribution of residual values due to the scattering nature of the "mixed" RRV, these problems could be solved by identifying the appropriate list of IPVs and selecting the proper model from the 21 models defined in Table 7.

The non-linear and mixed models adopted in this study have the potential to serve as an alternative to existing methods in predicting concrete strength based on mixture design variables with alternative or waste materials involved. Generally, the non-linear and mixed models achieved higher accuracy than the linear regression approach in predicting concrete strength as proved in this study and by the comparison with existing studies (Table 7 versus Table 1). Also, as shown in Table 11, compared to ANN and other data mining methods, the best-fit non-linear and mixed models proposed in this research achieved similar prediction performance based on both the numerical and relative input systems. These also have the advantage of being less time-consuming in model creation and allowing the analysis of individual materials' effects on concrete properties at different curing ages.

Table 11
Existing studies that used advanced or non-linear models to predict concrete strength.

Study	Independent variables	Adopted models	Sample size	R^2 result	Findings
Saridemir et al. [5]	BFS, curing age, PC, water, and aggregate	ANN and FL	284	As high as 1.00 for ANN and 0.991 for FL	ANN and FL had strong potential in predicting the CS.
Atici [6]	Proportion of BFS, FA, curing age, rebound number	MRA and ANN	135	As high as 0.98 for ANN and 0.90 for MRA	ANN outperformed MRA in predicting CS. However, MRA has its advantages.

Omran et al. [7]	Amount of individual ingredients in concrete mixture design including PLC, FA, and LWA	Nine different data mining methods including ANN, M5P model tree, etc.	144	Highest R^2 value achieved (0.984) by the additive regression method	Four regression tree models improved the prediction accuracy. Other three advanced models achieved higher accuracy, but the time required for building and training these models may be a restraint.
Chithra et al. [8]	Amount of cement, fine and coarse aggregates, nano silica, slag, and superplasticizer	MRA and ANN	264	Around 0.670 for MRA and close to 1.0 for ANN	MRA was found with lower accuracy and less satisfactory in meeting other statistical requirements (Durbin-Watson value) compared to ANN.
This study	Concrete-mixture-design-based inputs in both numeric and relative systems	MRA including linear, non-linear, and mixed models	144	Over 0.950 achieved in both numerical and relative input systems	Both non-linear and mixed models achieved better performance than the linear approach using both input systems. They can also statistically quantify alternative or waste materials' effects on concrete properties at different curing ages.

6. Conclusions

The regression analysis in this study provided a quantitative tool to predict concrete strength purely based on mixture-design-related variables and curing age. This statistical tool has the advantages of being easy-to-use and low-cost, not requiring extensive lab testing and large datasets, and achieving high degree of reliability. The non-linear and mixed models proposed in this research enrich the existing statistical modeling approach, which was usually limited to the linear regression method. The non-linear and mixed models could also serve as an alternative approach to existing data mining methods (e.g., ANN). The major findings of this study are summarized below:

- The proposed non-linear and mixed regression models achieved higher accuracy compared to the linear method in predicting concrete strength using the same concrete mixture variables and datasets. The best-fit models reached comparably high R^2 values (ranging from 0.934 to 0.955) as some data mining techniques. It is recommended that these models be applied to the datasets of previous studies to examine their potential in improving prediction accuracy.
- Using a comprehensive set of variables from the concrete mixture design, including both conventional and alternative/waste materials, was found to be viable in predicting the strength of sustainable concrete. It is expected that the list of IPVs could still be expanded

when more alternative materials from the cementitious or aggregate parts are added into concrete mixtures.

- Using the two input systems (i.e., numerical and relative) yielded highly consistent R^2 values in predicting concrete strength when the same RRV was adopted in the regression models. However, for practical reasons, the more straightforward numerical input system would be preferable as it allows the direct use of variable values from concrete mixture design. Conversion would be needed for the relative input system.
- Shortening IPVs based on internal correlation analysis would only cause a small performance loss when using the best-fit models to predict concrete strength. The corresponding statistical values (e.g., *t*, *p*, and coefficient) would better quantify the effect of each remaining IPV on the target RRVs. This research recommends keeping IPVs related to the studied material(s) (e.g., IPVs related to PLC, FA and LWA in this study) in the shortlist. As a result, the effects of studied material(s) on concrete properties could be properly quantified.
- The non-linear and mixed statistical models could simplify the prediction of concrete strength at certain curing age (e.g., Day 3, 7, or 90). They could also provide the statistical guide on the effects of alternative or waste materials on concrete mechanical properties as concrete age increases.

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