

# NON LINEAR AND LINEARIZED COMBINATION COEFFICIENTS FOR MODAL PUSHOVER ANALYSIS

P. Colajanni<sup>1</sup>, P. Cacciola<sup>2</sup>, B. Potenzzone, N. Spinella<sup>3</sup>, G. Testa<sup>1</sup>

<sup>1</sup> Dipartimento di Ingegneria Civile, Aerospaziale, Ambientale, e dei Materiali, Palermo University, Viale delle Scienze, Ed.8, 90128 Palermo, Italy

<sup>2</sup> School of Environment and Technology, University of Brighton, Lewes Road, BN24GJ Brighton, UK

<sup>3</sup> Dipartimento di Ingegneria Civile, Messina University, Piazza Pugliatti 1, Messina, Italy

**SUMMARY:** *current design practice and seismic codes tend to assess seismic demand of buildings by Non linear Static Analysis (NSA), based on the evaluation of the pushover curve. Earlier non-linear static analysis procedure estimate the response peak value by evaluating the push-over curve adopting a distribution of invariant forces proportional to the fundamental vibration mode. In order to include the effect of higher modes several multi-modal push-over analysis procedures were proposed in literature. In the most famous of these, namely Modal Pushover Analysis (MPA), nodal response peak values are obtained by combination of “modal” responses by the traditional SRSS or CQC methods: the use of the CQC rule is mandatory for irregular plane frames or spatial structures possessing modes with close natural frequencies. In order to take into account the actual characteristics of modal oscillators, the use of pertinent cross correlation coefficients defined for non-linear systems is required. In this paper the accuracy of correlation coefficients for linear systems in predicting the statistical correlation of hysteretic oscillator responses is investigated by a parametric analysis by Monte Carlo simulation. Furthermore, new correlation coefficients, determined through a pertinent statistical linearization are introduced, and the results provided by the classical and the proposed approach for two illustrative irregular plane and spatial frame are compared with non-linear time history analysis results, showing the effectiveness of the new procedure.*

**KEYWORDS:** *Pushover analysis, CQC, MPA, correlation coefficients*

## 1 Introduction

From the beginning of the century, performance-based seismic engineering is gaining prominence in current design codes (FEMA 356, 2000; EUROCODE 8, 2003) and is increasing popular in practice, especially through advances in displacement-based design and assessment methods. Despite the fact that Nonlinear Response History Analysis (NRHA) is the most accurate and powerful tool for the evaluation of the seismic demand, the large computational effort required and the sensitivity of the results to structure and ground motion modelling make

---

Corresponding author: Piero Colajanni, Department of Civil Engineering, Aerospace, Environmental and Materials, University of Palermo, Italy.

Email: [piero.colajanni@unipa.it](mailto:piero.colajanni@unipa.it)

the method unsuitable for engineering practice. Thus, Non-linear Static Procedure (NSP), often referred to as Push Over Analysis (POA), is now recommended by codes as the reference method for seismic design and performance assessment.

POA was proposed in different forms by Saiidi and Sozen (1981), and Fajfar and Gaspersic (1996). In standard POA (FEMA 356, 2000) the seismic demand is computed forcing the structure by monotonically increasing lateral forces, with an invariant distribution, until a predetermined target displacement is reached. The target displacement is chosen by modification of the spectral displacement corresponding to the period of the SDOF, equivalent to the first modal oscillator, by coefficients able to take the level of inelasticity, hysteresis shape and P- $\Delta$  effect into account. By tracing the sequence of yielding on the resisting members, and consistently updating the structure stiffness matrix, POA aims at evaluating the whole structure capacity curve, as well as inter-storey drifts, forces and local inelastic deformations in the structural members induced by the design seismic action.

Let us stress that the non-linear static response of a yielding structure is very sensitive to the load distribution, and therefore the choice of the appropriate load pattern is a key issue. Extension of the traditional linear elastic static analysis to the non-linear field leads to assuming an inverted triangular force pattern or proportional to the first mode. Nonetheless, invariant force distributions are not able to take into account the redistribution of inertia forces due to yielding, and the associate change in the mode shape. Moreover, force distribution and displacement pattern related to the fundamental period of vibration do not account for the contribution of higher modes.

To overcome the former limitation, and with the aims of bounding the likely distribution of inter-story drifts and local ductility demands along the height of the structure, seismic codes require that the analysis is performed assuming two different seismic force patterns, e.g. the first mode shape and the uniform load pattern. The latter pattern aims at reproducing the expected inertia force distribution consistent with the formation of a soft storey at the base of the structure, characterized by a constant displacement at each storey. More refined approaches include the so-called adaptive pushover procedure (see, e.g. Bracci et al, 1997, Colajanni and Papia 1998, Gupta and Kunnath, 2000) where, unlike conventional pushover analysis, the progressive stiffness degradation and the change of modal characteristics are taken into account, attempting to follow the time-variant distributions of inertia forces closer.

The limitation related to the use of fundamental mode properties only gives satisfactory predictions of seismic demand mostly when dealing with in plane and in elevation regular low and medium-rise structures, for which higher mode effects are minimal. Thus, multi-modal push-over procedures have been proposed including the effect of higher modes (e.g. Sasaki et al., 1998; Moghadam, 1998; Chopra and Goel, 2002; Kunnath, 2004, Park et al. (2007), Abbasnia et al. (2013)). Earlier attempts were done by Sasaki et al. (1998), that proposed a method to identify the sensitivity of the structure to higher mode effects in pushover analysis, Moghadam (1998) formulated a procedure where the seismic demand due to the individual terms in the modal expansion of earthquake forces is determined by pushover analysis. The final response of the building is obtained by algebraic summation of the response of the “modal” pushover analyses, taking into account the value of the participation factors, and neglecting that the “modal” oscillators do not reach the maximum displacement at the same instant.

Chopra and Goel (2002), following the same approach, proposed a method known as Modal Pushover Analysis (MPA), showing that the modal demand pertinent to the first two or three modes, combined through the classical modal combination rules, for regular plane frames

provided a generally good estimate of the total seismic demand, inter-story drifts and plastic hinge location. They also recognized that the main approximation of the methods lies on the assumption that the non-linear behaviour of each individual mode is decoupled from each other, ignoring the contribution of other modes in the formation of plastic hinges. Nonetheless, the authors in the same paper give evidence that, if the structure is designed ensuring the activation of a global collapse mechanism, the assumption is realistic in various cases of engineering interest, even if the structure is forced in the inelastic range. In a subsequent paper, Chintanapakdee and Chopra (2003) stressed that another source of approximation arises from evaluating the total seismic demand by superposition of modal response with the classical modal combination rules that have been derived for linear classically damped systems.

In order to take into account the effect of the higher mode in the load vector, Kunnath (2004) proposed to evaluate the seismic demand by envelope of the result obtained by multiple analyses in which the load pattern are evaluating by summing and subtracting two or more components of the modal expansion of the load vector suitably scaled. Park et al. (2007) proposed their modal combination coefficients, calculated from a comprehensive set of elastic time history analyses. In Kreslin and Fajfar (2011) a procedure that takes into account the higher mode effect in elevation was proposed, and generalized to both in elevation and in plane irregular structures in Kreslin and Fajfar (2012). In the latter procedure, the quadratic combination rule is applied for evaluation of the linear dynamic system response, which shape is utilized for the definition of two amplification coefficients used for scaling the response obtained by conventional pushover analysis. More recently, Abbasnia *et al.* (2013) stressed that the recently developed adaptive pushover procedures by which effect of higher modes as well progressive damage accumulation are taken into account, suffer from the quadratic modal combination rule, in which the sign reversal of load vector in higher mode are neglected. They tried to solve this drawback by proposing a new effective modal mass combination rule to be implemented in a new procedure that require multiple pushover analysis.

Poursha (2009, 2015) proposed a method in which the demand is obtained by the envelope of the response of 2 or 3 analysis, respectively. The first one is an analysis with conventional profile of time-invariant forces, while in the second one a load profile proportional to the first modal shape is applied until reaching a first target displacement; the additional load increases come distributed according to the second modal shape, until reaching the final target displacement. These increases are acting on the structure in the loading, stresses and displacements configuration obtained at the end of the first analysis segment. The technique is called "Consecutive Modal Pushover". Recently, Menun, Reyes and Chopra (2015) stressed the role of the errors caused by peak factor assumptions in modal combination rules.

The use of the combination rules to calculate the peak response of inelastic systems is questionable. Specifically, the basic assumption in deriving the well-known SRSS and CQC combination rules relies on modelling the structural response as a zero-mean Gaussian stationary process. Clearly, the response of the non-linear hysteric modal oscillator cannot be considered as Gaussian even if the input is modelled or assumed as a Gaussian process. Therefore, the use of traditional modal combination rules requires a preventive approximation of the seismic response of hysteretic modal oscillator by equivalent linearized ones.

In this paper, a theoretically consistent approach for combination of modal response in MPA, able to incorporate the main features of the CQC rule in linear elastic analyses, is proposed. Specifically, each individual modal oscillator is linearized by the stochastic averaging technique (Roberts and Spanos, 1997) assuming the Rayleigh probability density

function for the amplitude response process. As a consequence, the equivalent damping and stiffness are determined using the procedure described by Lutes and Sarkani (2004) for white noise input. Finally, the CQC rule for MPA is used for combining the modal responses of the linearized system, by using correlation coefficients pertaining to linearized modal oscillators. They can be quite different from the linear ones as both the equivalent damping and stiffness are strongly affected by the nonlinear hysteretic behaviour, enlarging the range in which the CQC rule instead of the SRSS is required. In order to highlight the range of values for which the use of proposed correlation coefficients can significantly improve the accuracy of the MPA, the equation of motion of the modal hysteretic systems is recast in a nondimensional form, and a parametric analysis is performed by Monte Carlo simulation. Finally, the results provided by classical and modified MPA, for an illustrative irregular plane frame, are compared with NRHA results showing the effectiveness of the proposed approach.

## 2 Modal Pushover Analysis

Let us briefly summarize the basic steps of the MPA method proposed by Chopra and Goel (2002). The response of a non-linear multistorey system subjected to base ground excitation is governed by the following equation of motion,

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{f}_s(\mathbf{U}, \text{sign}\dot{\mathbf{U}}) = -\mathbf{M}\mathbf{1}\ddot{U}_g(t) \quad (1)$$

where  $\ddot{\mathbf{U}}, \dot{\mathbf{U}}, \mathbf{U}$  are acceleration, velocity, and displacement vectors, respectively;  $\mathbf{M}, \mathbf{C}$  are mass and damping matrices,  $\mathbf{f}_s$  is the vector containing the lateral forces at  $N$ -th floor levels, that depend on the history of the displacement,  $\mathbf{1}$  is the influence vector, and  $\ddot{U}_g(t)$  is the ground acceleration. According to the MPA approach, the right side of equation (1) can be expressed as effective earthquake forces reflecting the distribution of inertia forces  $\mathbf{s}_n$

$$\mathbf{M}\mathbf{1} = \sum_{n=1}^N \mathbf{s}_n = \sum_{n=1}^N \Gamma_n \mathbf{M}\Phi_n \quad (1)$$

where  $\Phi_n$  is the  $n$ -th natural vibration mode determined by considering the linear initial state for which  $\mathbf{f}_s(\mathbf{U}, \text{sign}\dot{\mathbf{U}}) = \mathbf{K}\mathbf{U}$ , and  $N$  is the number of the degrees of freedom of the system. It follows that  $\Phi_n$  is determined by the solution of the following eigenproblem

$$\mathbf{K}\Phi = \mathbf{M}\Phi\Omega^2 \quad (3)$$

$\Omega^2$  being the spectral matrix listing the natural frequencies  $\omega_n$  of the initial linear system and

$$\Gamma_n = \frac{L_n}{M_n}; \quad L_n = \Phi_n^T \mathbf{M}\mathbf{1}; \quad M_n = \Phi_n^T \mathbf{M}\Phi_n; \quad (2)$$

thus, the effective earthquake forces can be expressed as:

$$\mathbf{p}_{\text{eff}}(t) = \sum_{n=1}^N \mathbf{p}_{\text{eff},n}(t) = \sum_{n=1}^N -\mathbf{s}_n \ddot{U}_g(t). \quad (3)$$

Interestingly, Chopra and Goel (2002) showed that if the inelastic multistory system is subjected to the individual component  $\mathbf{p}_{\text{eff},n}(t)$ , the dynamic response is roughly proportional to the  $n$ -th linear vibration mode  $\Phi_n$ . Thus, let us consider the governing equation of the inelastic system subjected to the individual contribution of the  $n$ -th mode to  $\mathbf{p}_{\text{eff}}(t)$

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{f}_s(\mathbf{U}, \text{sign}\dot{\mathbf{U}}) = -\mathbf{s}_n\ddot{U}_g(t). \quad (4)$$

By applying the following coordinate transformation

$$\mathbf{U}(t) = \Phi\mathbf{Q}(t) \quad (7)$$

the  $n$ -th modal equation becomes:

$$\ddot{Q}_n + 2\zeta_n\omega_n\dot{Q}_n + \frac{F_{sn}(\mathbf{Q}, \text{sign}(\dot{\mathbf{Q}}))}{M_n} = -\Gamma_n\ddot{U}_g(t) \quad (8)$$

where  $\zeta_n$  is the damping ratio. Clearly, equation (8) is not a proper modal equation due to the nonlinearity of the system. Nevertheless, in the following, the term “modal” will be used for emphasizing that the forcing term is proportional to individual linear mode. Furthermore, it is noted that modal equations are coupled by means of term  $F_{sn}$ , that is the projection of the non-linear resisting force in the modal subspace.

According to the MPA approach the modal equations are assumed decoupled determining the non-linear term by classical push over analysis. Specifically, setting

$$Q_n(t) = \Gamma_n D_n(t) \quad (5)$$

the decoupled modal equation is cast in the form

$$\ddot{D}_n + 2\zeta_n\omega_n\dot{D}_n + \frac{\tilde{F}_{sn}(D_n, \dot{D}_n)}{L_n} = -\ddot{U}_g(t) \quad (6)$$

in which  $\tilde{F}_{sn}$  is the bilinear hysteretic term determined forcing the structure with an invariant force distribution

$$\mathbf{s}_n^* = \mathbf{M}\Phi_n \quad (7)$$

and approximating the capacity curve of the representative global hysteretic behavior of the non-linear structure by an equivalent bilinear system, having the same capacity of absorbing energy of the actual structure. The procedure for the evaluation of the equivalent bilinear hysteretic system is described in detail in FEMA-356 (2000).

The base shear-roof displacement ( $V_{bn} - U_{rn}$ ) relationship is considered as a capacity curve. The co-ordinates of the idealized curve are converted into the corresponding SDOF co-ordinates, ( $D_n, \tilde{F}_{sn} / L_n$ ) as follows:

$$D_n = \frac{U_{rn}}{\Gamma_n \Phi_{rn}}, \quad \frac{\tilde{F}_{sn}}{L_n} = \frac{V_{bn}}{M_n^*} \quad (8)$$

where  $M_n^*$  is the effective modal mass, given by:

$$M_n^* = L_n^2 / M_n. \quad (9)$$

The yield value of  $D_n$  and  $\tilde{F}_{sn} / L_n$  are

$$D_{ny} = \frac{U_{my}}{\Gamma_n \Phi_m}, \quad \frac{F_{sny}}{L_n} = \frac{V_{bny}}{M_n^*}. \quad (10)$$

Therefore, the peak value of the  $n$ -th mode,  $U_{m0}$ , is given by

$$U_{m0} = \Gamma_n \Phi_{m0} D_{n0} \quad (11)$$

where  $D_{n0}$  is the peak of the inelastic response determined by the solution of equation (10), or from the corresponding design inelastic spectrum. Finally, the peak nodal response is determined by combining each individual modal peak value according to the well-known classical modal combination rules, such as SRSS (Rosenblueth 1951) or CQC (Wilson et al, 1981). For modes possessing close natural frequencies, the CQC method is usually preferred leading to the following equation for the maximum nodal response

$$U_{r,CQC} = \sqrt{\sum_{j=1}^m \sum_{k=1}^m \rho_{jk} U_{rj0} U_{rk0}} \quad (12)$$

where  $m < N$  is the number of modes retained.

By examining equation (16) two major drawbacks appear quite evident. First of all, each individual modal oscillator behaves nonlinearly. It follows that even if the ground motion is modeled as a Gaussian white noise process, the response will not be Gaussian. As a consequence, equation (16) cannot be applied rigorously. Moreover, the correlation coefficients used for combining each individual peak modal contribution in the classical CQC, under the hypothesis of white noise input, are a function of the linear modal frequencies and damping ratios. It is noted that due to non linearity, both the modal frequency and the damping ratio should reflect the variation from the initial linear behavior of the system and thus they have to be suitably modified.

### 3 Correlation coefficients for Modal Pushover Analysis

In previous sections it has been emphasized that a crucial point for applying the modal pushover analysis is how the contributions of each individual non-linear modal maximum are combined. According to the classical CQC method, the combination of modes is based on the definition of proper correlation coefficients defined on the basis of the random response of linear systems. In this section the approximations introduced in the MPA for combining the modal peaks are scrutinized. A study on the random response of bilinear hysteretic oscillators and pertinent correlation coefficients is initially conducted. New correlation coefficients defined on the basis of the Stochastic Averaging method are herein proposed. Furthermore, a parametric study shows the range of values in which the classical correlation coefficients determined for the linear case should be properly replaced.

#### 3.1 Formulation

Let us consider the modal bilinear oscillator determined through the MPA approach governed by equation (10). According to the classical CQC method, seismic action is modelled as a zero-mean Gaussian white noise process with unilateral power spectral density  $G_{\dot{U}_g} \equiv G_{W^0}$ . In order

to investigate the range value of the parameters for which the CQC method, defined for the linear case, fails to combine the maxima pertinent to the MPA method, let us consider the non-dimensional displacement

$$X_n = \frac{D_n}{\sigma_{D_n}} \quad (13)$$

where  $\sigma_{D_n}$  is the standard deviation of response  $D_n$  in which the restoring force  $\tilde{F}_{sn}(D_n, \dot{D}_n)/L_n = \omega_n^2 D_n$  in the linear initial state is retained. That is,

$$\sigma_{D_n} = \sqrt{\frac{\pi G_{W^0}}{4\zeta_n \omega_n^3}} \quad (14)$$

Introducing the following non-dimensional parameters

$$\alpha = \frac{\zeta}{\sqrt{1-\zeta^2}}; \quad \tau = \omega\sqrt{1-\zeta^2}t \quad (15)$$

equation (10) can be recast in the form (Ditlevsen and Bognar, 1993)

$$\ddot{X}(\tau) + 2\alpha\dot{X}(\tau) + F_s(X, \dot{X}; \tau) = W(\tau) \quad (16)$$

in which the index  $n$  has been omitted for simplicity's sake.  $W(\tau)$  is the scaled excitation with unilateral power spectral density

$$G_W = 4\alpha(1+\alpha^2). \quad (17)$$

The bilinear hysteretic systems (Fig. 1) is composed of a hysteretic Jenkins' element, consisting of a linear spring with stiffness  $(1-\nu)(1+\alpha^2)$  in series with a Coulomb or slip damper which has maximum allowable force  $(1-\nu)(1+\alpha^2)X_y$ , and a linear spring with stiffness  $\nu(1+\alpha^2)$ ,  $\nu$  being the hardening ratio.

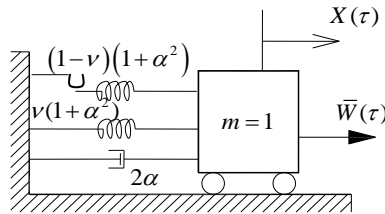


Figure 1. Non dimensional bilinear oscillator

Furthermore, the non-dimensional yield value  $X_y$  is given by

$$X_y = \frac{D_y}{\sigma_D} \quad (18)$$

According to the MPA approach, the nodal response is determined by the superposition of the modal nonlinear peaks through the cross correlation coefficients defined by equation

$$\rho_{jk} = \frac{E[X_j X_k]}{E[X_j^2] E[X_k^2]} \quad (19)$$

$E[\bullet]$  being the mathematical expectation and  $X_i$  ( $i = j, k$ ) is the solution of the following set of equations

$$\begin{aligned} \ddot{X}_j(\tau) + 2\alpha\dot{X}_j(\tau) + F_{s,j}(X_j, \dot{X}_j; \tau) &= W(\tau) \\ \ddot{X}_k(\tau) + 2\alpha\beta_{jk}\dot{X}_k(\tau) + \beta_{jk}^2 F_{s,k}(X_k, \dot{X}_k; \tau) &= W(\tau) \end{aligned} \quad (20)$$

where  $\beta_{jk} = \omega_j / \omega_k$  is the ratio between the modal frequencies related to the first branch of the bilinear curves. For simplicity's sake the correlation coefficients used in the classical CQC in conjunction with the MPA approach refer to the initial linear state, according to the equations

$$\rho_{jk} = \frac{8\sqrt{\zeta_j \zeta_k} (\zeta_j + \beta_{jk} \zeta_k) \beta_{jk}^{3/2}}{(1 - \beta_{jk}^2)^2 + 4\zeta_j \zeta_k \beta_{jk} (1 + \beta_{jk}^2) + 4(\zeta_j^2 + \zeta_k^2) \beta_{jk}^2} \quad (21)$$

and for  $\zeta_j = \zeta_k = \zeta$

$$\rho_{jk} = \frac{8\zeta^2 (1 + \beta_{jk}) \beta_{jk}^{3/2}}{(1 - \beta_{jk}^2)^2 + 4\zeta^2 \beta_{jk} (1 + \beta_{jk})^2} \quad (22)$$

As the above correlation coefficients are determined for the linear case, they are independent of the system yielding strength and can be considered as a rough approximation of the exact coefficients determined by equations (23) and (24). Moreover, it has to be emphasized that, as the traditional CQC method is determined under the hypothesis that the response is modeled as a Gaussian stationary process, equation (16) should not be applied directly in MPA. In this paper, a linearization strategy is proposed as a vehicle for combining the modal maxima. If each individual non-linear oscillator is linearized first, it follows that, due to the Gaussianity of the input process the modal response will be Gaussian. Therefore, as the nodal response is the result of the superposition of Gaussian processes, also the nodal response will be Gaussian too. The equivalent linear oscillator can be determined by various strategies (Roberts and Spanos, 1999) (Elishakoff and Colajanni 1998). In this paper, the equivalent damping and stiffness are determined through a version of the stochastic averaging technique (see, e.g. Lutes and Sarkani, 2004). Specifically, a pseudo-harmonic behavior of the response process is assumed

$$X(\tau) = A(\tau) \cos(\gamma_{eq} \tau + \mathcal{G}(\tau)) \quad (23)$$

where the amplitude  $A(t)$  and the phase  $\mathcal{G}(t)$  are slowly varying with respect to time;  $\gamma_{eq}$  is the nondimensional equivalent frequency. Therefore, the bilinear hysteretic term is replaced in the harmonic balance sense by the equation

$$F_s(X, \dot{X}; \tau) = \nu(1 + \alpha^2)X(\tau) + \eta_1 \dot{X}(\tau) + \eta_2 \ddot{X}(\tau) \quad (24)$$

Accordingly, the linearized equation (20) is cast in the form

$$\ddot{X}(\tau) + (2\alpha + \eta_2) \dot{X}(\tau) + (\nu(1 + \alpha^2) + \eta_1) X(\tau) = W(\tau) \quad (25)$$

The coefficients  $\eta_1$  and  $\eta_2$  are determined by simultaneously solving the following set of equations,



$$\begin{aligned}
 \gamma_{n,eq} &= \sqrt{\nu(1+\alpha^2) + \eta_1} \\
 \sigma_x^2 &= \frac{2\pi\alpha(1+\alpha^2)}{(2\alpha + \eta_2) [\nu(1+\alpha^2) + \eta_1]} \\
 \eta_1 &= \frac{2}{\sigma_x^2} \left( \int_0^{X_y} (1-\nu)(1+\alpha^2)u^2 \frac{u}{\sigma_x^2} \exp\left[-\frac{u^2}{2\sigma_x^2}\right] du \right. \\
 &+ \left. \int_{X_y}^{\infty} \frac{(1-\nu)(1+\alpha^2)u}{\pi} \cos^{-1}\left[\frac{u-2X_y}{u}\right] - \frac{2(1-\nu)(1+\alpha^2)}{\pi} \left(\frac{u-2X_y}{u}\right) \sqrt{X_y(u-2X_y)} \frac{u}{\sigma_x^2} \exp\left[-\frac{u^2}{2\sigma_x^2}\right] du \right) \\
 \eta_2 &= \left(\frac{2}{\pi}\right)^{1/2} \frac{(1-\nu)(1+\alpha^2)X_y}{\gamma_{eq}\sigma_x} \left(1 - \Psi\left(\frac{X_y}{\sigma_x}\right)\right) \tag{26}
 \end{aligned}$$

determined assuming the amplitude  $A(\tau)$  Rayleigh distributed. Therefore, the equivalent correlation coefficients are given by either equation (25) or equation (26), replacing the above evaluated equivalent damping and stiffness. It has to be emphasized that the correlation coefficients between the responses of two bilinear oscillators are function of the following non-dimensional parameters: the ratio,  $\beta_{jk}$ , between the modal frequencies related to the first branch of the bilinear curves; the hardening ratios  $\nu_i$  ( $i = j, k$ ); the yield values  $X_{iy}$  ( $i = j, k$ ), and the damping ratios  $\zeta_i$  ( $i = j, k$ ), the latter related to the non-dimensional parameter  $\alpha$  by the means of equation (19). In the following section the influence of the above nondimensional parameters on the correlation coefficient trend is investigated by a pertinent parametric study.

### 3.2 Parametric study

In this section the cross correlation coefficients are scrutinized for various scenarios of the above defined fundamental nondimensional parameters. Specifically, the classical correlation coefficients defined for the linear case are compared with the “exact” ones determined by Monte Carlo simulation through equations (23) and (24), and the “linearized” correlation coefficients, determined through the stochastic averaging technique. The results are conveniently represented in term of the yield strength reduction factor  $q$  (Chintanapakdee and Chopra, 2003), defined by equation

$$q = \frac{X_{el}}{X_y} \tag{27}$$

$X_{el}$  being the peak response determined assuming an elastic linear behavior. Let us stress that for both  $q \rightarrow 1$  and for  $q \rightarrow \infty$  the response behaves linearly, according to the stiffness of first or second branch of the hysteretic restoring force, respectively. In Figure 2a), the correlation coefficients, determined by a pertinent Monte Carlo Simulation (MCS) and Equivalent Linearization (EL) conducted on the bilinear oscillators for various values of the ratio  $\beta_{jk}$ , are

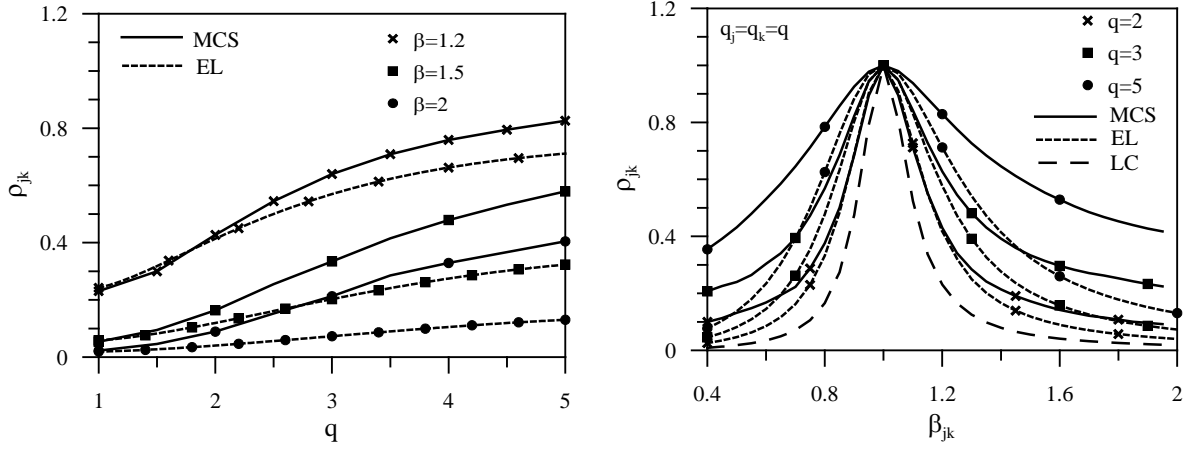


Figure 2. Non linear correlation coefficients for  $v_j = v_k = v = 0.30$ ,  $\zeta_j = \zeta_k = \zeta = 0.05$

shown as a function of the parameter  $q$ . As the parameter  $q$  increases the values of the correlation coefficients increase accordingly, manifesting for higher values of  $q$  variations of about 100% with respect to the linear case determined for  $q \rightarrow 1$ . As evidenced by Chopra and Goel (2002), the MPA could inaccurately predict the nodal demand for strong nonlinearity and for higher excitation level. The main source of error is recognized to be related to the decoupling of the modal equations. Interestingly, a further source of error is herein manifested by the analysis of the correlation coefficients that are strongly different from the usual coefficients determined for the linear case as the parameter  $q$  increases. Furthermore, the stochastic averaging technique reliably predicts the trend of the correlation coefficients as a function of the parameter  $q$ , clearly, approximating the “exact” ones accurately only for low values of nonlinearity.

In Figure 2b), it is assumed that  $q = q_j = q_k$ ; as expected, the classical correlation coefficients are quite different from the exact coefficients, determined by a pertinent Monte Carlo study even for low levels of the parameter  $q$ . It is noted that the differences increase as the ratio  $\beta_{jk}$  between two modal frequencies increases. For generic frames this feature shows that if the contribution of higher modes is not negligible the greater is the parameter  $\beta_{jk}$ , the greater is the error for evaluating the cross modal contribution. On the other hand, appreciable differences can be found for low values of the parameter  $\beta_{jk}$  (i.e. approaching the unity). Indeed, in this case, generally related to plane irregular frames or space frames, the correlation coefficients approaches the unity and the contribution of the cross modal terms become comparable with the direct ones. The linearized correlation coefficients provide an improvement in accuracy as they are also dependent on the yielding value. Therefore, the higher the level of nonlinearity, the narrower is the range of the parameter  $\beta_{jk}$ , close to the unity, in which the linearized correlation coefficient approximates the exact coefficients accurately.

In Figure 3, the correlation coefficients for different values of the yielding level (i.e.  $q_j \neq q_k$ ) are shown. Interestingly, it can be noted that the value of  $\beta_{jk}$ , for which the correlation coefficient is maximum, is a function of the parameters  $q_i$  ( $i = j, k$ ).

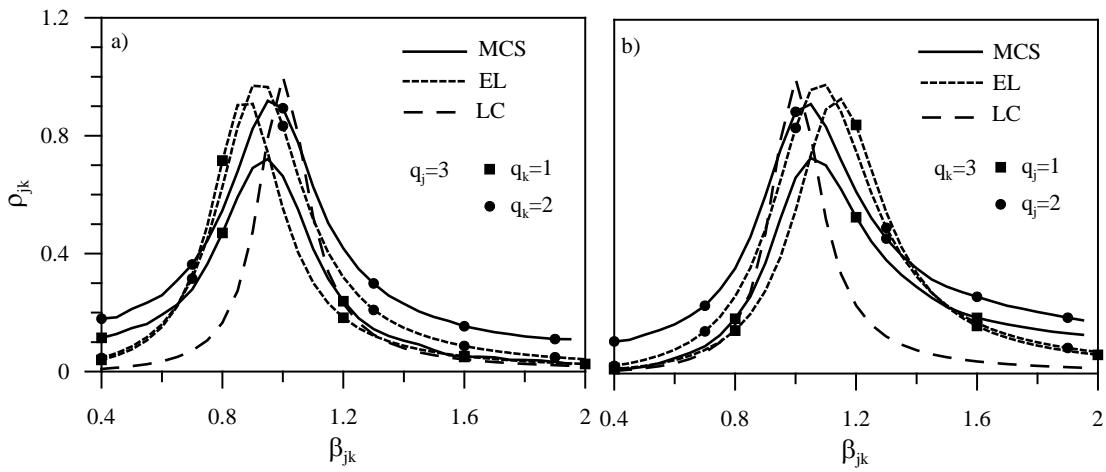


Figure 3. Non linear correlation coefficients for  $v_j = v_k = v = 0.30$ ,  $\zeta_j = \zeta_k = \zeta = 0.05$ ; a)  $q_j = 3$ , b)  $q_k = 3$

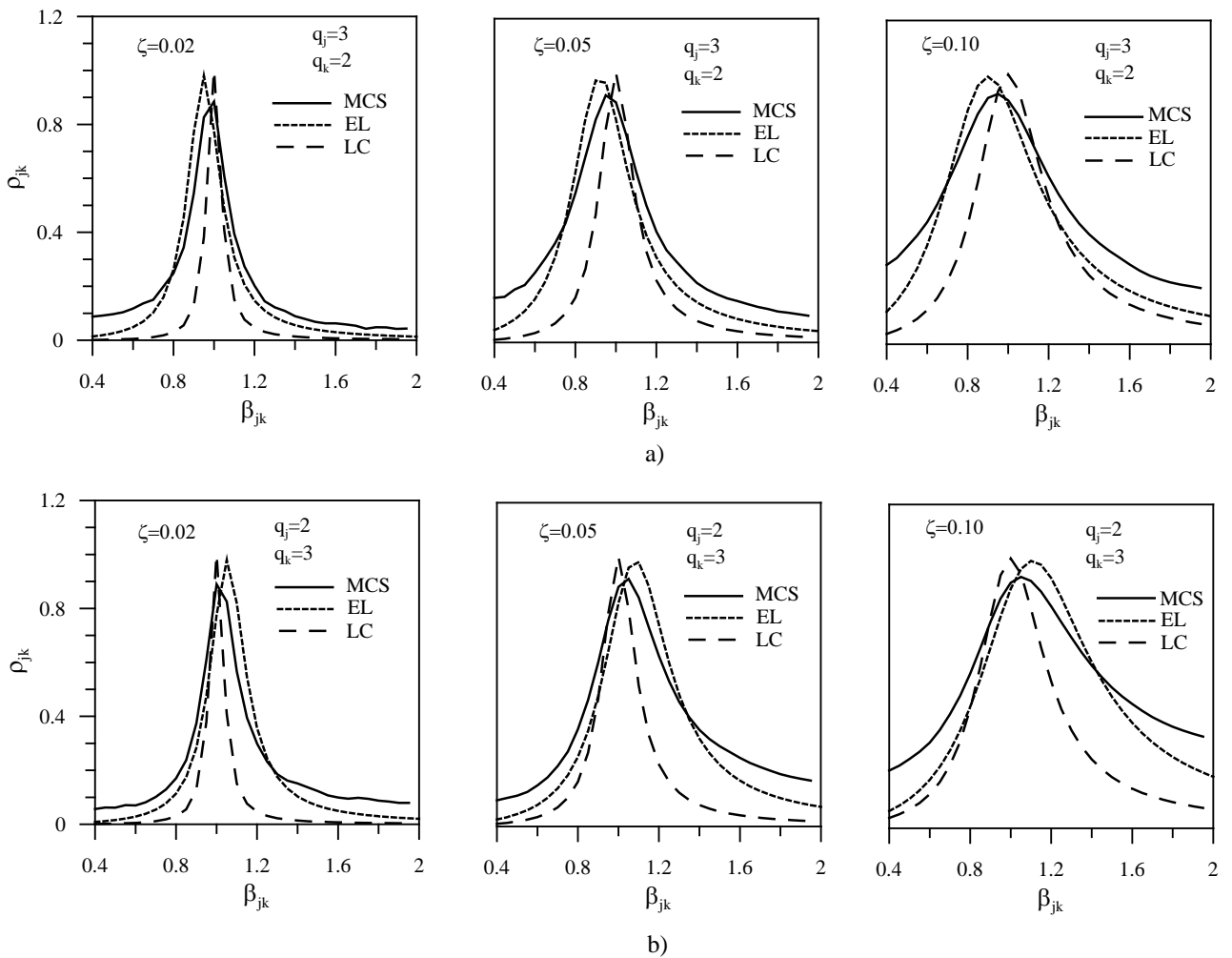


Figure 4. Non linear correlation coefficients for  $v_j = v_k = v = 0.30$ ,  $\zeta_j = \zeta_k = \zeta$ ; a)  $q_j = 3$   $q_k = 2$ , b)  $q_j = 2$   $q_k = 3$

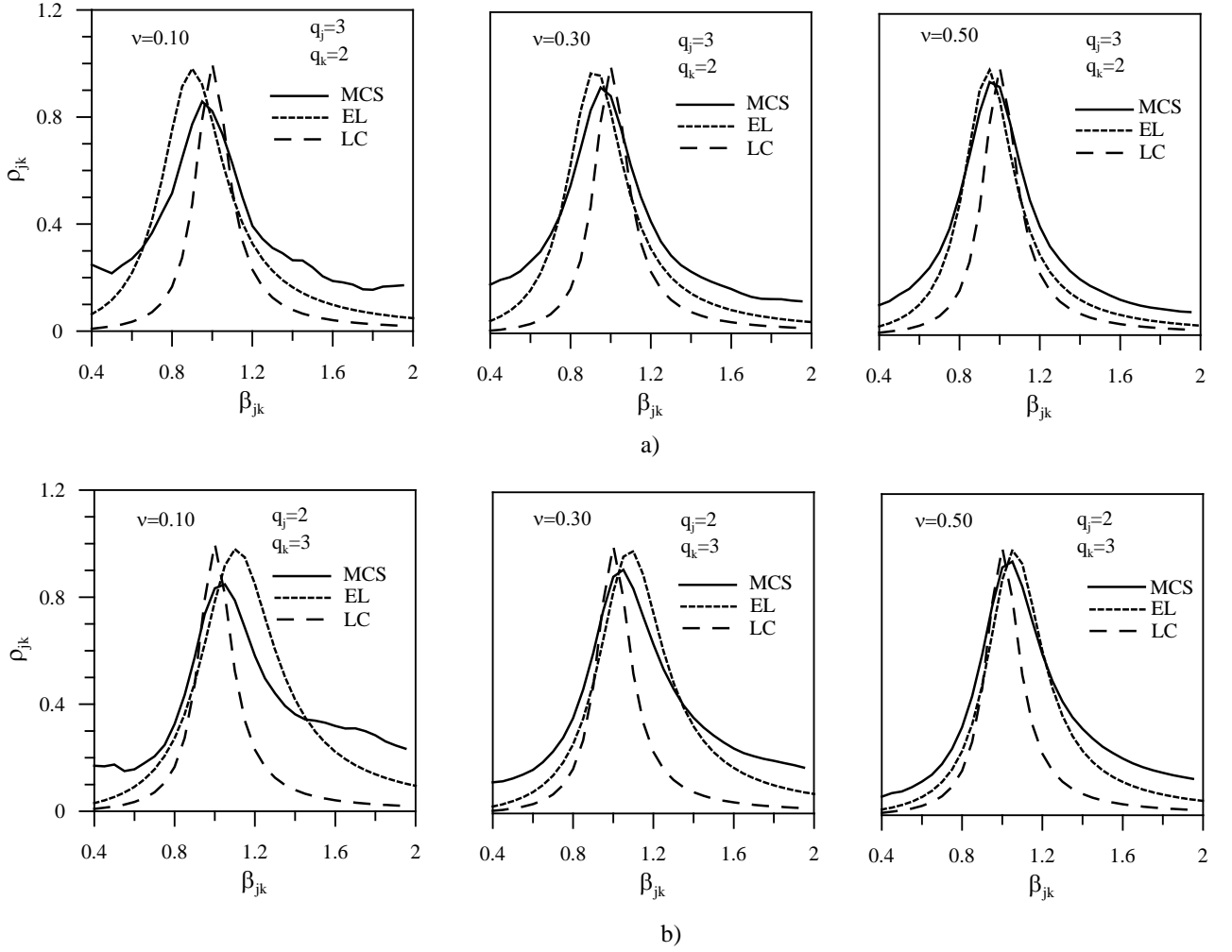


Figure 5. Non linear correlation coefficients for  $\nu_j = \nu_k = \nu$ ,  $\zeta_j = \zeta_k = 0.05$ ; a)  $q_j = 3$   $q_k = 2$ , b)  $q_j = 2$   $q_k = 3$

In this case, the linear correlation coefficients determined by equation (25) are quite inaccurate, even for low levels of excitation. On the other hand, the linearized correlation coefficients manifest the same trend as the non-linear ones, maintaining a satisfactory accuracy for low values of the parameter,  $q_i$ . It has to be emphasized that for  $q_j \neq q_k$  the non-linear correlation coefficients, as well as the nonlinearized ones, are no longer symmetric. That is

$$\rho_{jk} \neq \rho_{kj}. \quad (28)$$

Clearly, this property has to be considered for combining the modal contribution. The influence of the damping  $\zeta$  and hardening ratio  $\nu$  is shown in Figures 4 and 5.

Remarkably, for various values of the damping ratios appreciable differences between the classical and exact correlation coefficients can be noted. Furthermore, owing to the more reliable assumption of pseudo-harmonic behaviour of the response, the lower the damping ratio  $\zeta$  the more the equivalent linearization accurately approximates the exact one. The influence of the stiffness ratios  $\nu$  is lastly investigated. For  $\nu \rightarrow 0$  the bilinear oscillator tend to an elastic-perfectly plastic behaviour. For  $\nu = 0$  the random response is highly non Gaussian manifesting a non-stationary behaviour deviating appreciably from the linear case (see e.g.

Caddemi and Colajanni, 1999). Therefore, the more the hardening ratio  $\nu$  increase the more the equivalent linearization accurately approximates the exact one.

## 5 Numerical applications

### Plane Frame

This section is concerned with the analysis of the response of a four-storey shear type steel frame with an appendage, subjected to the El Centro (1940) earthquake, studied by Chopra (1995). The structure idealizes generic one-bay frames equipped of a flexible appendage, modelling light structures such as an advertising billboard, an antenna, a small housing for mechanical equipment or the like. The author recognized that this system brings out special response features manifesting, in the linear case, the influence of the cross term in applying the CQC rule.

The structure possess lumped masses at the first four floors  $m_j = m = 45.34 \text{ kN/g}$  ( $j = 1, \dots, 4$ ) and at the fifth floor, relative to the appendage,  $m_5 = 0.01m$ . The lateral stiffness of each of the first four stories is  $k_j = k = 3957 \text{ kN/m}$  ( $j = 1, \dots, 4$ ), while the fifth-story stiffness is  $k_5 = 0.0012k$ , ( $j = 1, \dots, 4$ ). The damping ratio of the first two modes is equal to 5%.

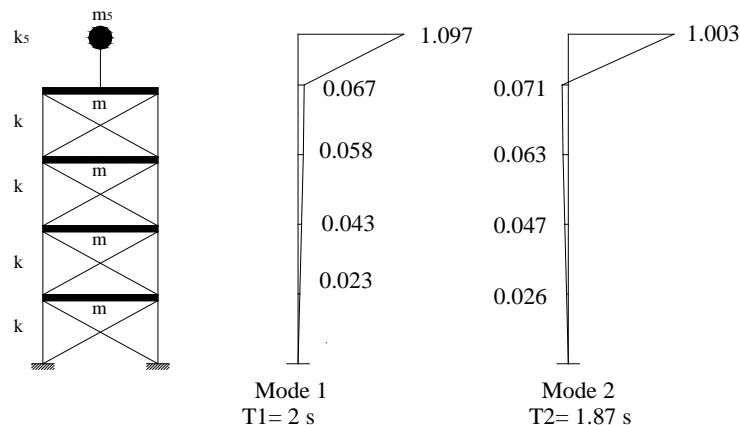


Figure 6. Structural scheme, natural period and modes of vibration of building with appendage

The first two modal shapes  $\Phi_n$  ( $n = 1, 2$ ) along with the corresponding natural periods  $T_n$  are shown in Figure 6. Being the appendage very light and very flexible the first two modal shapes manifest localized large deformations at the top floor. Furthermore, the participation mass ratios are 45.7% for the first mode and 43.7% for the second, respectively. In order to investigate the post-elastic behaviour of the structure and the capability of the MPA for evaluating the total seismic demand, and the inter-storey drifts, the structure is designed so that the first four floors remain in the elastic state, and only the fifth one, having a shear resistance of 1.17 KN, yields. Following the procedure described in previous sections MPA is applied. The “modal” pushover curves, in which the base shear  $V_b$  is divided by the total weight, and the roof displacement  $U_r$  by the total height are shown in Figure 7.

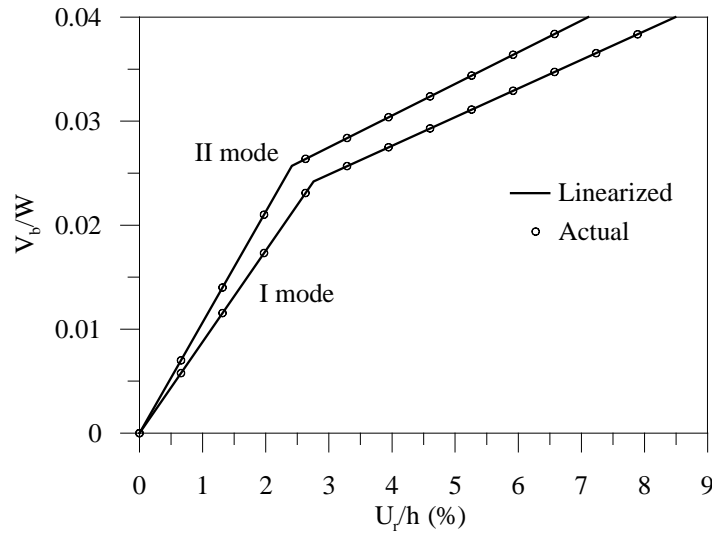


Figure 7. “Modal” Pushover Curves of I Mode and II Mode

It can be noted that the perfect coincidence between the actual and the idealized bilinear curves is due to the concentration of the inelastic deformation in the column of the fifth storey. The idealized bilinear modal oscillator hardening ratios are  $\nu_1 = 0.318$  and  $\nu_2 = 0.286$ , and the yield strength reduction factors are  $q_1 = 3.12$  and  $q_2 = 2.97$ , respectively.

The corresponding non-linear correlation coefficients obtained by MCS, corresponding to the two frequency ratios  $\beta_{12}=0.83$  and  $\beta_{21}=1.20$  are  $\rho_{12} = 0.93$  and  $\rho_{21} = 0.92$ , about 75 % greater than their linear counterpart; Moreover, by using the stochastic averaging technique the values of the linearized correlation coefficients are  $\rho_{12} = 0.94$  and  $\rho_{12} = 0.93$ .

A comparison between the absolute values of the peaks response determined by MPA and the benchmark solution obtained by NRHA, both for roof displacement and inter-storey drift, is calculated. The peak responses are calculated by both SRSS and CQC modal combination rules. Furthermore the CQC method is applied by considering the three correlation coefficient values, evaluated by MCS, equivalent linearization (EL) and linear case (LC) according to the hypothesis of white noise input process.

It has to be emphasized that MPA allows two alternative strategies for evaluating the response peaks of modal oscillators  $D_{n0}$ , namely the nonlinear response history analysis (NRHA), and the response spectrum technique. Both of them have been applied and pertinent results are shown in Tables 1 and 2, respectively.

The use of SRSS rule gives great error in the evaluation of the response peaks, because of the close frequencies; by contrast the use of CQC rule decreases the errors. In the Table 1 a share of error is due to the approximation of the decoupling of non-linear modal equations in MPA. In Table 2, where the peak response of the modal oscillator are evaluated by the response spectrum, unexpectedly, the errors are smaller than their counterpart in Table 1; this is due to the fact that the share of the errors due to the evaluation of the modal oscillator response peak values tend to cancel with the previous mentioned one. However, it can be remarked that the use of non-linear correlation coefficients strongly enhanced the effectiveness of MPA for all the considered cases, and the equivalent linearization provides a very good estimation of the non-linear coefficients

Table 1. Error in peak response combination in MPA: modal peaks estimated by NRHA

	SRSS $\varepsilon$ (%)	CQC-LC $\varepsilon$ (%)	CQC-EL $\varepsilon$ (%)	CQC-MCS $\varepsilon$ (%)
Top displacement	160	47	24	19
Top storey drift	168	48	30	25

Table 2. Error in peak response combination in MPA: modal peaks estimated by response spectrum

	SRSS $\varepsilon$ (%)	CQC-LC $\varepsilon$ (%)	CQC-EL $\varepsilon$ (%)	CQC-MCS $\varepsilon$ (%)
Top displacement	218	78	11	5
Top storey drift	228	81	16	9

*Spatial Frame*

The effectiveness of the use of non-linear correlation coefficients in the combination of the modal contributions in the MPA is shown here for the moment resisting steel spatial frame (MRF) analysed in Goel and Chopra (2004), represented schematically in elevation in Figure 8a) and in plan in Figure 8b).

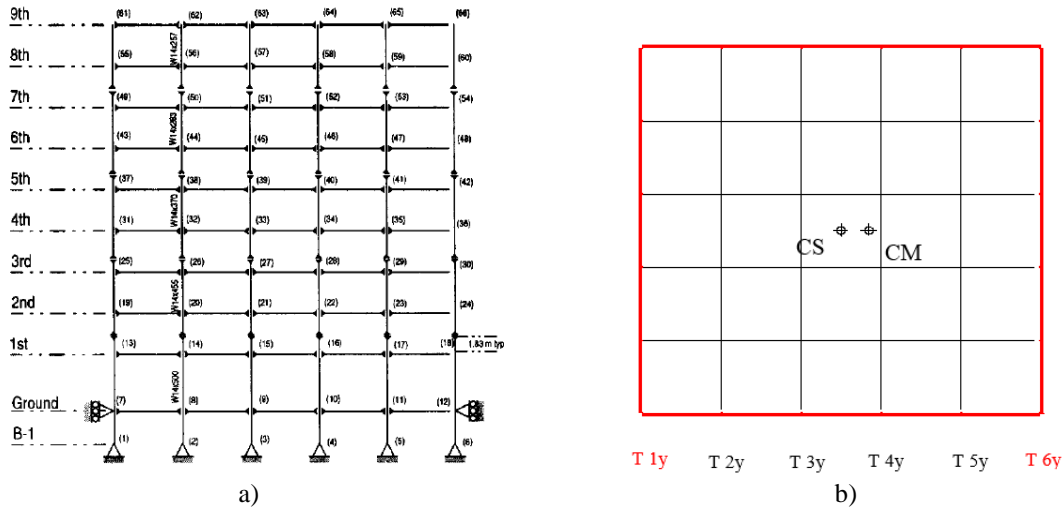


Figure 8 .Numerical example a) elevation, b) plan

The structure is realized with six frames having the height of 31.17 m (9 levels) for each of the two main directions, and it has a square plan of side 45.75 m. The mass of the first level is  $10.01 \times 10^6$  kg, instead in the intermediate floors is equal to  $9.89 \times 10^5$  kg, while the roof has mass  $1.07 \times 10^6$  Kg. The structure is regarded as irregular in plant due to an eccentricity along the x direction between the centre of masses (CM) and the geometric centre (coinciding with the stiffness centre CS) equal to 4.75 m. The resistance of the steel beams is 248 Mpa, while that of the pillars is 345 Mpa. The beams are made out of the following ASTM standard profiles: first and second level W 36x160, from the third to the sixth 36x135 W, seventh W 30x99, 27x84 W eighth, ninth W 24x68. The pillars are made out of: first and second level W 14x500,

from the third to the sixth W 14x455, 14x370 W seventh, eighth 14x 283 W, ninth 14 W x257. Input acting in the x direction is modelled by a set of 10 natural accelerograms, chosen for to be compatible with the response spectrum for soil type B proposed by Eurocode 8, characterized by peak ground acceleration  $a_g=0.5 g$ . The Figure 9b) shows the capacity curves for the individual modes obtained assuming as a control point the top floor mass centre; the curves are stopped at the target displacement, and they show the well-known difference between the stiffness of the individual modes.

Figure 10a) compares the trend along transversal y direction of the top floor maximum displacements in the seismic input x direction, estimated through different assumptions in MPA. Initially, in order to enucleate the source of errors arising from the approximation of the decoupling of non-linear modal equations, from that due to ineffective use of modal combination rule, the benchmark is evaluated by the "Uncoupled Modal Response History Analysis (UMRHA) (Chopra 2002). In UMRHA the response is evaluated by time history combination of modal contributions (inelastic oscillators characteristics obtained from pushover analysis for each modes considered significant), according the displacement pattern obtained for each mode at the end of the pushover analyses. Then MPA analyses were conducted with different assumptions for the correlation coefficients (CCs) and compared, namely neglecting CCs (SRSS), CCs evaluated for linear modal oscillators subjected to white noise input (CQC-LWN) Eq.(25), linear modal oscillators subjected to spectrum compatible process (Cacciola et al. 2004) (CQC-LSPC), and non linear oscillator response evaluated by MCS with SPC process (CQC-MCSPC).

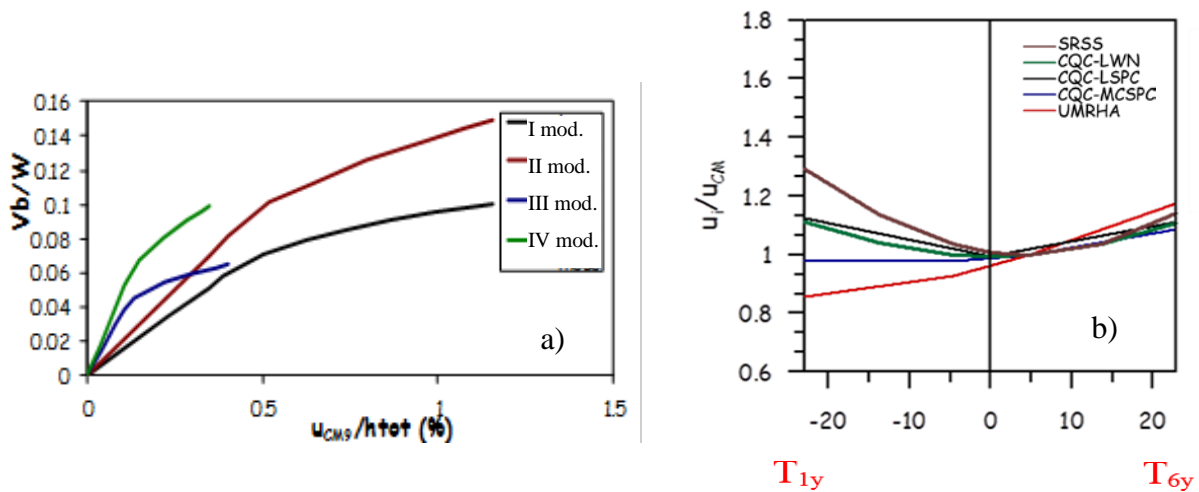


Figure 9. a) modal pushover curves; b) peak roof displacements along the x direction

The curves show that the values of the maximum displacement estimated for the end frame  $T_{6y}$  placed at the flexible side of the structure, provided by the different modelling of the correlation coefficients are very similar, and reproduce with good approximation the result of UMRHA. By contrast, Figure 9 shows that at the stiff side ( $T_{1y}$  frame) the approximations connected with the use of SRSS are coarse. Only the use of CQC with correlation coefficients evaluated taking into account the hysteretic characteristic of the modal oscillators and the actual energy frequency content of the input (CQC-MCSPC), are able to reproduce the results provided by UMRHA. The unsatisfactory results provided by linear spectrum-compatible correlation coefficients (CQC-LSPC) shows that the nonlinear characteristics of modal



oscillator are decisive in reproduce the actual response. Figure 10a) shows the displacements along the height of the structure for the T6y frame, while Figure 10b) shows the curves for the homologous inter-storey drift. Lastly, Figure 11a) and Figure 11b) show the trend along the height of the maximum floor and inter-story displacements, respectively, for T1Y frame place at the stiff side of the structure. The figures show that the above considerations can be extended to the story displacement or inter-story drift at any storey of the frame.

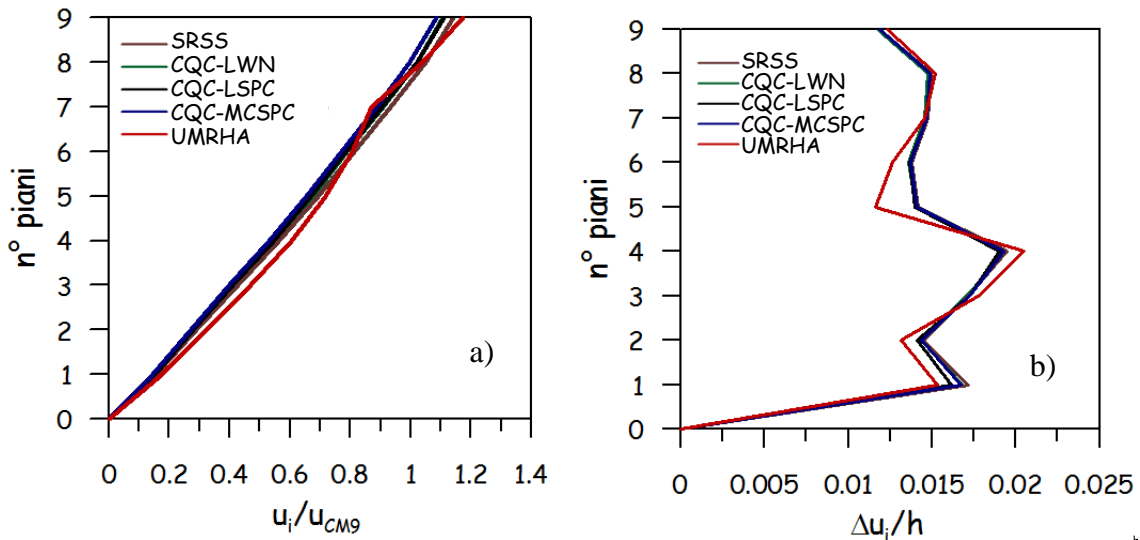


Figure 10. T6y frame: a) maximum storey displacements; b) maximum storey drift

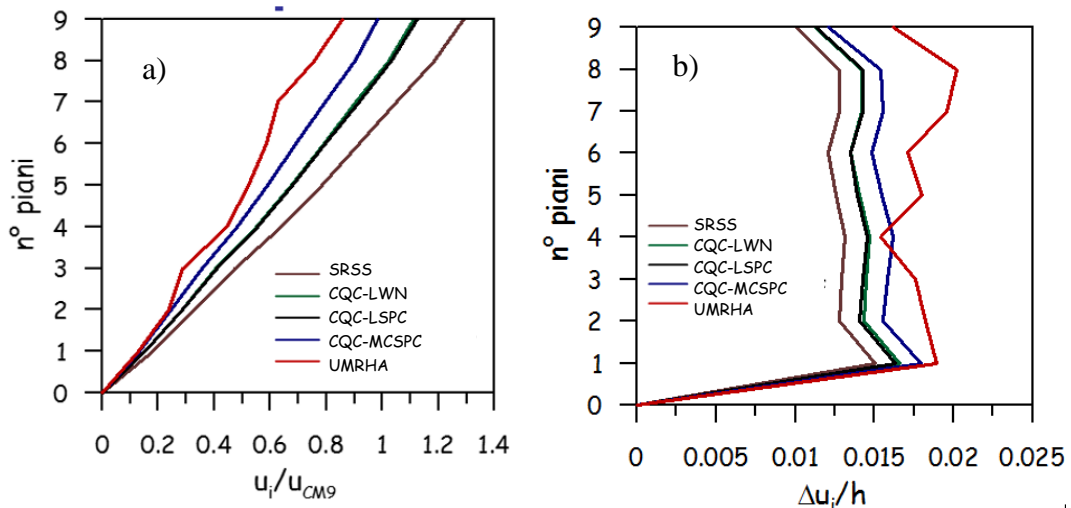


Figure 11. T1y frame: a) maximum storey displacements; b) maximum storey drift.

## Conclusion

One of the key issues in multimodal pushover analysis is the combination of the peaks of the modal responses. The traditional modal combination rules derived for linear systems cannot be

directly extended to hysteretic oscillator. Specifically, the correlation coefficient appearing in the CQC rule, required for taking into account the response correlations of structure having close natural frequencies, like irregular plane frame or space framed buildings, are ineligible for modal combination in MPA.

A procedure for extension of the traditional CQC rule to MPA, based on the linearization of the hysteretic modal oscillators by the Stochastic Averaging method, has been proposed in the paper. New correlation coefficients able to take in to account the hysteresis properties of the modal oscillator has been defined. Moreover, by a non-dimensional formulation of the equations governing the response of the modal oscillators, it has been stressed that assuming a white noise representation of the seismic input, the correlation coefficients are a function of the frequency ratio of the two modal oscillator in the initial elastic state, and their damping ratios and yield strength reduction factors.

A parametric analysis of the correlation coefficients for hysteretic systems has shown that: - the greater the yield strength reduction factor, the greater the difference between traditional correlation coefficients and those pertaining to hysteretic systems; - the difference is noticeable in the entire range of the modal oscillator frequency ratio and damping ratios; appreciable values of the correlation coefficients can be found for well separate modal oscillator frequencies; - the smaller is the hardening ratio, the more the difference increases. These results lead one to conclude that in the MPA the range in which it is necessary the application of the CQC rule is larger than in conventional elastic analysis.

Moreover, the proposed method for evaluation of correlation coefficients for hysteretic system by the stochastic averaging method is effective in capture the trend of the actual coefficients in the entire parameter ranges. However the method, the more the frequency ratio of the two modal oscillator and yield strength reduction factors approach the unity, the more is accurate. Further investigations are needed in order to explored the correlation coefficients behaviour for hysteretic system when input representation consistent with response spectrums are considered.

## References

- Abbasnia R, Davoudi A T and Maddah M M (2013), "An adaptive pushover procedure based on effective modal mass combination rule," *Engineering Structures*, 52: 654-666.
- Bosco M, Ferrara G, Ghersi A, Marino E and Rossi P (2015), "Seismic assessment of existing r.c. framed structures with in-plan irregularity by nonlinear static methods," *Earthquake and structures*: 401-422.
- Bracci JM, Kunnath SK and Reinhorn AM (1997)," Seismic performance and retrofit evaluation of reinforced concrete structures," *Journal of Structural Engineering*, **123**(1):2-10.
- Cacciola P., Colajanni P., Muscolino G. (2004) Combination of modal responses consistent with seismic input representation, *Journal of Structural Engineering-ASCE*, Vol.130, pp.47-55. ISSN: 0733-9445, ISI:000187786000005
- Chintanapakdee C and Chopra AK (2003), "Evaluation of modal pushover analysis using generic frames," *Earthquake Engineering and Structural Dynamics*, **32**:417-442.

- Chopra AK (1995), *Dynamics of Structures*, Prentice-Hall, Inc., Englewood Cliffs, N.J.
- Chopra A K and Goel RK (2002), “A modal pushover analysis procedure for estimating seismic demands for buildings,” *Earthquake Engineering and Structural Dynamics*, **31**: 561-582.
- Colajanni P and Papia M (1998), “A push-over analysis method for axially deformable cross-braced frames with dissipative devices,” *ISET Journal of Earthquake Technology*, **35**(4): 263-286.
- De Stefano, Tanganelli M and Viti S (2015), “Torsional effects due to concrete strength variability in existing buildings” *Earthquake and structures*: 379-399.
- Ditlevsen O and Bogner L (1993), “Plastic displacement distributions of the Gaussian white noise excited elasto-plastic oscillator,” *Probabilistic Engineering Mechanics*, **8**: 209-231.
- Elishakoff I, and Colajanni P. (1998), Booton's problem re-examined *Journal of Sound and Vibration*, Vol. 210, N°5, Academic Press Limited, London, pp.683-691.
- European Committee for Standardization (2003), Eurocode 8: *Design of structures for earthquake resistance-Part 1: General rule, seismic actions and rules for building*, Brussels.
- Fajfar P and Gaspersic P (1996), “The N2 method for the seismic damage analysis of RC buildings,” *Earthquake Engineering and Structural Dynamics*, **25**: 31-46.
- Fema 356 (2000), *Prestandard and commentary for the seismic rehabilitation of buildings*, Federal Emergency Management Agency, Washington D.C.
- Gupta B and Kunnath SK (2000), “Adaptive Spectra-Based Pushover Procedure for Seismic Evaluation of Structures,” *Earthquake Spectra*, **16** (2): 367-391.
- Kreslin M and Fajfar P (2011), “The extended N2 method taking into account higher mode effects in elevation,” *Earthquake engineering and Structural Dynamics*, **40**: 1571-1589.
- Kreslin M and Fajfar P (2012), “The extended N2 method considering higher mode effects in both plan and elevation, *Bull. Earthquake Eng.*, 2012, 10, 695-715
- Kunnath S (2004), ”Identification of Modal Combinations for nonlinear static analysis of building structures,” *Computer-Aided Civil and Infrastructure Engineering*, **19**: 246-259.
- Lutes LD and Sarkani S (2004), *Random Vibration: Analysis of Structural and Mechanical System*, Elsevier, Burlington.
- Menun C, Reyes J.C. Chopra A.K. (2015) Errors caused by peak factor assumptions in response-spectrum based analyses, *Earthquake Engng Struct. Dyn.* 2015; 44:1729–1746.
- Moghadam AS (2002), “A pushover procedure for tall buildings,” *Proceedings of the 12th European Conference on Earthquake Engineering*, cd-rom, London, England, Elsevier Science Ltd.
- Moghadam AS and Ashtiany M (2015), “Collapse response assessment of low-rise buildings with irregularities in plan,” *Earthquake and structures*: 49-71.

- Nezhad ME and Poursha (2015), "Seismic evaluation of vertically irregular building frames with stiffness, strength, combined stiffness and strength and mass irregularities," *Earthquake and structures*: 353-373.
- Park H.G., Eom T, Lee H. (2007). Factored modal combination for evaluation of earthquake load profiles. *J Struct Eng, ASCE*, 133(7):956–68.
- Panyakapo P (2014), "Cyclic Pushover analysis procedure to estimate seismic demands for building," *Engineering Structures*, 66: 10-23.
- Poursha M, Khoshnoudian F and Moghadam AS (2009), "A consecutive modal pushover procedure for estimating the seismic demands of tall building," *Engineering Structures*, 31: 591-599.
- Roberts JB and Spanos PD (1999), *Random Vibration and Statistical Linearization*, Mineola, New York.
- Sasaki KK, Freeman SA and Paret, TF (1998), "Multi mode pushover procedure(MMP)-a method to identify the effects of higher modes in a pushover analysis," *Proceedings of the Sixth U.S. National Conference on Earthquake Engineering, Earthquake Engineering Research Institute*, Oakland, California, 12 pages.
- Saiidi M and Sozen MA (1981), "Simple non linear seismic analysis of R/C structures," *Journal of the Structural Division*, **107**(ST5), 937-51.