

# A Fuzzy Logic Approach to Stochastic 1D Site Response Analysis accounting for Soil Uncertainties

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**Abstract:** Site response analysis, namely the analysis of the wave propagation of shear waves through a soil deposit, requires the specification of the input ground motion and the dynamic characterization of the soil deposit. While the stochastic approach is commonly used for modelling seismic excitation, the use of probability density functions for describing the soil properties is consistent only when precise informations based on a large amount of data from soil surveys are available. Conversely, a non-probabilistic approach based on fuzzy set theory would be more appropriate for dealing with uncertainties that are just expressed by vague, imprecise, qualitative, or incomplete information and supplied by engineering judgement. In this paper, a hybrid fuzzy-stochastic 1D site response analysis approach for dealing with soil uncertainties defined as convex normal fuzzy sets is addressed. Zadeh's extension principle, in combination with an efficient implementation of the Differential Evolution Algorithm is used for global minimization and maximization. Results are presented as fuzzy median value of the largest peaks of the peak ground acceleration at the surface by considering four types of soil classified in accordance with the European seismic building code.

## 1 Introduction

Site response analysis aims to predict the influence of the local site effects on the characteristics of the earthquake motion. The most widely used technique is the study of the one-dimensional amplification of vertically propagating waves by solving the dynamic wave equation in the frequency domain with equivalent linear elastic soil properties. While the traditional practice deals the seismic input motion as the only source of uncertainty by considering it as random, owing to the complexity of its intrinsic structure, soil characterization manifests various sources of uncertainties due to the soil spatial variability and to the dispersion of the soil parameters that should be taken into account. Although site property variabilities are sometime assumed having a normal or lognormal distribution, e.g. [3], due to the geologic process, the natural spatial variability of the soil can be relevant resulting in a strong variation of the properties even over small distances; soil properties maps, generated from soil surveys, do not provide sufficient information about soil deposits and rock formation. Furthermore, due to the large amount of data required to estimate the parameters for the dynamic geotechnical characterization of the soil deposit, the use of probability density functions for all of them becomes inconsistent; a probabilistic model dealing with these uncertainties requires supplying greater knowledge than that gained from actual experience. [2] showed, for geotechnical systems, the high sensitivity in calculating the failure probability when different distributions obtained by fitting the same input data from laboratory tests are used.

The genuine lack of knowledge or imprecision in the definition of a property, in addition to the dispersion of the data caused by systematic measurement errors, fluctuations and sample disturbance, determine unavoidable uncertainty of an epistemic nature. Converse to aleatory uncertainty, epistemic uncertainty can eventually be reduced through the collection of more and better data. In order to avoid misleading representations, several approaches alternative to the probabilistic method, referred to as non-probabilistic methods, have been developed. In particular, fuzzy set theory [6] can be applied for dealing with non-random, incomplete, imprecise information as well as linguistic vagueness, namely the use of natural linguistic information in engineering judgement knowledge, for classifying generic class of soil (e.g. soft, medium, rigid) or the soil deposit type (e.g. class A-B-C-D according to the EN 1998-1). Fuzzy set theory uses the concept of possibility in which a fuzzy set  $\tilde{A}$ , is described as a class of objects with a continuum of grades of membership  $\mu_{\tilde{A}}$ , ranging from  $\alpha = 0$ , i.e. the object does not belong to the set, to  $\alpha = 1$ , i.e. the object completely belongs to the set. In this paper, the fuzzy logic approach is used for dealing with uncertainties of the main parameters involved in the site response analysis, i.e. the shear wave modulus  $G_0$ , the soil unit density  $\rho_0$ , the critical damping ratio  $\xi$ , and the depth of the soil deposit  $h$ . The membership function of the fuzzy output  $\tilde{a}_{PGA}(\tilde{h}, \tilde{G}_0, \tilde{\rho}_0, \tilde{\xi})$  representing the fuzzy median value of the largest peaks of the free field acceleration at the top surface, as a function of the fuzzy parameters  $\tilde{G}_0$ ,  $\tilde{\rho}_0$ ,  $\tilde{h}$  and  $\tilde{\xi}$ , is obtained by the Zadeh's extension principle, in combination with an efficient implementation of the Differential Evolution Algorithm for global minimization and maximization [4]. In section 2 we describe the one-dimensional site response analysis problem in terms of its associated stochastic equation. Section 3 briefly describes the basic concepts and tools of fuzzy numbers and fuzzy calculus, including the application of the differential evolution (DE) algorithm to the extension principle. The fuzzy stochastic site response model and the results of its application to four different soil deposits are illustrated in section 4. Conclusions are drawn in section 5.

## 2 Stochastic 1D site response analysis problem

The stochastic equation of the one-dimensional (1D) site response analysis problem is addressed in this section by considering the seismic motion uncertain, in particular random. The site response analysis aims to evaluate the effects of the local soil conditions on the amplitude and frequency content of the seismic motion that propagates through the soil deposit during an earthquake event. By applying the approach based on Random Vibration Theory, the stochastic process of the ground motion acceleration  $\ddot{U}_g(\omega)$ , modelled as a zero-mean stationary Gaussian stochastic process fully described by the knowledge of its power spectral density function, denoted by  $S_{\ddot{U}_g \ddot{U}_g}(\omega)$ , where  $\omega \geq 0$  is the circular frequency and each superscript dot indicates time differentiation, is propagating vertically through the soil deposit from the bedrock ( $z = z_{bed}$ ) to the ground surface acceleration through the following expression in the frequency domain:

$$S_{\ddot{U} \ddot{U}}(\omega) = |H(\omega)|^2 S_{\ddot{U}_g \ddot{U}_g}(\omega) \quad (1)$$

where  $H(\omega)$  is the transfer function of the soil deposit representing the ratio between the acceleration  $\ddot{U}(\omega)$  at the surface and the acceleration  $\ddot{U}_g(\omega)$  at the bedrock computed by solving the one-dimensional soil amplification problem described in the frequency domain by the following dynamic equation:

$$G(1 + 2i\xi) \frac{d^2 U(\omega, z)}{dz^2} = \rho \omega^2 U(\omega, z) \quad (2)$$

where  $z$  is the depth from the ground surface,  $\rho$  is the soil density,  $G$  is the shear stiffness modulus of the soil,  $\xi$  is the critical damping ratio, and  $i$  is the imaginary unit. It is worth noting that the dependency of the soil properties,  $\rho$ ,  $G$ ,  $\xi$  in the transfer function  $H(\omega)$  has been

omitted. In order to take into account the nonlinearities of the soil properties, a discretization of the soil deposit in more layers with constant properties and a equivalent linear methodology is applied. Statistical quantities of the response are thus derived from the stochastic Eq. (1) for each  $i$ -th layer of local coordinate  $\zeta_i$ ; in particular, the characteristic acceleration  $X_{\ddot{U}}$  is computed as the fractile of order  $p$  (usually the median, i.e.  $p = 0.5$ ) of the distribution of maxima through the first crossing problem defined as follows:

$$X_{\ddot{U},i}(T_s, p, \zeta_i) = \eta_{\ddot{U},i}(T_s, p, \zeta_i) \sqrt{\lambda_{0,\ddot{U},i}} \quad (3)$$

where  $T_s$  is the time length of the stationary part of the signal;  $\eta_{\ddot{U},i}$  are the peak factors determined by the relation obtained by [5];  $\lambda_{0,\ddot{U},i}$  is the zero-order response spectral moment of the acceleration, expressed as

$$\lambda_{0,\ddot{U},i} = \int_0^\infty S_{\ddot{U},i}(\omega) d\omega, \quad (4)$$

It should be stressed that the dependency of the spectral moment of Eq. (4) to the system parameters has been omitted for brevity. Therefore, objective of this paper is the investigation of  $X_{\ddot{U}}$  from Eq. (3) when the soil properties are uncertain but not random.

### 3 Fuzzy approach for accounting soil parameter uncertainty

In this section, a fuzzy logic approach for stochastic 1D site response analysis, when soil parameters are uncertain, is established. In our soil amplification problem, in combination with the random nature of the input motion, sources of uncertainty include variability in material properties, such as the shear elastic modulus, the unit density, and the damping ratio as well as geometric boundaries as the thickness of the soil deposit. These uncertainties are mainly caused by measurement errors, sampling disturbance and/or incomplete knowledge about soil description and the use of natural language for classifying the ground type (e.g. soft, soft-to-firm, stiff, very stiff) or the soil type (e.g., as classified by the Unified Soil Classification System). In this context, Fuzzy sets theory [6] has been shown to be effective for dealing with the epistemic nature of these uncertainties (e.g see [1]). Especially when evidences do not allow a probability interpretation of the data sets, fuzzy logic is a reasonable approach for capturing the vagueness meaning of their properties; moreover, contrary to the use of interval analysis, where only upper and lower bounds are assigned to each parameter, the fuzzy sets provide further information about the grade of possibility (or possibility distribution) on the interval.

#### 3.1 Fuzzy sets and Intervals

Given the soil property, or more generally, the system parameter  $A$ , its representation as a *fuzzy set*  $\tilde{A}$  over a given set (or space)  $\mathbb{X}$  of elements (the universe) is usually defined by its membership function

$$\mu_{\tilde{A}} : \mathbb{X} \longrightarrow [0, 1] \quad (5)$$

and a fuzzy (sub)set  $\tilde{A}$  of  $\mathbb{X}$  is uniquely characterized by the pairs  $(x, \mu_{\tilde{A}}(x))$  for each  $x \in \mathbb{X}$ ; the value  $\mu_{\tilde{A}}(x) \in [0, 1]$  is the membership grade of  $x$  to the fuzzy set  $\tilde{A}$ .

Our interest are fuzzy sets when the space  $\mathbb{X}$  is  $\mathbb{R}$  (unidimensional real fuzzy sets). Denote by  $\mathcal{F}(\mathbb{R})$  the collection of all the fuzzy sets over  $\mathbb{R}$ . Fundamental concept in fuzzy theory is the *level-sets* (or *level-cuts*) of its membership function); for  $\alpha \in ]0, 1]$ , the  $\alpha$ -level cut of  $\tilde{A}$  (or simply the  $\alpha$ -cut) is defined by

$$[\tilde{A}]_\alpha = \{x | x \in \mathbb{R}, \mu_{\tilde{A}}(x) \geq \alpha\} \quad (6)$$

. It can be related to the level of knowledge of the fuzzy set  $A$ , in which for increasing values of  $\alpha$ , the uncertainty of the system parameter decreases.

The  $\alpha$  – cuts of a fuzzy number or interval are non empty, compact intervals of the form

$$[\tilde{A}]_\alpha = [A_\alpha^-, A_\alpha^+] \subset \mathbb{R}. \quad (7)$$

If  $A_\alpha^- = \hat{A}^-$  and  $A_\alpha^+ = \hat{A}^+$ ,  $\forall \alpha \in [0, 1]$  we have a crisp interval or a crisp number (if, in addition,  $\hat{A}^- = \hat{A}^+$ ).

Any fuzzy number or interval  $\tilde{A} \in \mathcal{F}^1$  has the well known LR-representation (L for left, R for right), i.e. its membership function is of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} A^L(x) & \text{if } a \leq x \leq c \\ 1 & \text{if } c \leq x \leq d \\ A^R(x) & \text{if } d \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where  $a \leq c \leq d \leq b$ , the function  $A^L: [a, c] \rightarrow [0, 1]$  is non-decreasing with  $A^L(a) = 0, A^L(c) = 1$  and the function  $A^R: [d, b] \rightarrow [0, 1]$  is non-increasing with  $A^R(d) = 1, A^R(b) = 0$ . The interval  $[a, b]$  is the support and  $[c, d]$  is the core. If  $c = d$ , we obtain a fuzzy number. We refer to the functions  $A^L(\cdot)$  and  $A^R(\cdot)$  as the lower and upper sides of  $\tilde{A}$ , respectively.

### 3.2 Fuzzy Extension Principle

Elements of  $\mathcal{F}^1$  will be denoted by letters  $\tilde{A}, \tilde{B}, \tilde{C}$  and the corresponding membership functions by  $\mu_{\tilde{A}}, \mu_{\tilde{B}}$ , and  $\mu_{\tilde{C}}$ . Given two fuzzy numbers  $\tilde{A}, \tilde{B} \in \mathcal{F}^1$ , the four arithmetic operations are defined by the use of the Zadeh's extension principle ( $\circ \in \{+, -, \times, /\}$ ):

$$\mu_{\tilde{A} \circ \tilde{B}}(z) = \sup_{z=x \circ y} \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\}. \quad (9)$$

Consider the extension of a function  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  to a vector  $\tilde{\mathbf{A}} = (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)$  of  $n$  fuzzy numbers, with  $k$ -th component  $\tilde{A}_k \in \mathcal{F}^1$  given, in terms of  $\alpha$ -cuts, by  $[\tilde{A}_k]_\alpha = [A_{k,\alpha}^-, A_{k,\alpha}^+]$  for  $k = 1, 2, \dots, n$ .

Denote by  $\tilde{B} = \tilde{f}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)$  the corresponding fuzzy interval. For a continuous function  $f: \mathbb{R}^n \rightarrow \mathbb{R}$ , the  $\alpha$  – cuts  $[B_\alpha^-, B_\alpha^+]$  of the fuzzy extension  $\tilde{B}$  are obtained by solving the following box-constrained global optimization problems ( $\alpha \in [0, 1]$ ):

$$B_\alpha^- = \min \{f(x_1, x_2, \dots, x_n) | x_k \in [\tilde{A}_k]_\alpha, k = 1, 2, \dots, n\} \quad (10)$$

$$B_\alpha^+ = \max \{f(x_1, x_2, \dots, x_n) | x_k \in [\tilde{A}_k]_\alpha, k = 1, 2, \dots, n\}. \quad (11)$$

For general functions, we need to solve numerically the global minimization and maximization problems above; regarding the particular nature of our problem, a differential evolution (DE) Method is applied which adopted strategy is the following: *SPDE (Single Population DE procedure)*: start with the  $(\alpha = 1)$  – cut back to the  $(\alpha = 0)$  – cut so that the optimal solutions at a given level can be inserted into the "starting" populations of lower levels; use two distinct populations and perform the recombinations such that, during generations, one of the populations specializes to find the minimum and the other to find the maximum. In this paper, the procedure SPDE has been implemented using MATLAB ; a detailed description with extended computational results can be found in [4].

## 4 Numerical Analysis

Stochastic 1D site response analyses are carried out by considering uncertainty in the definition of the soil parameters described according to the fuzzy approach. Four different soil deposits

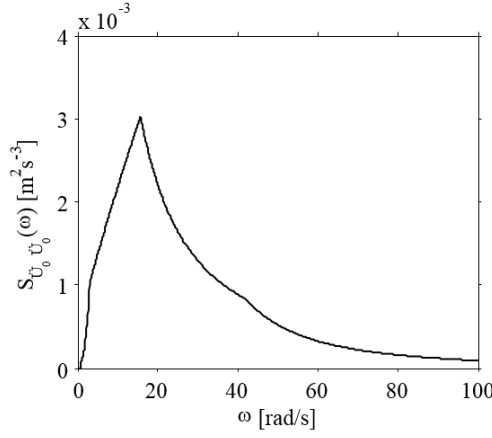


Figure 1: Power spectral density function defined at the outcrop bedrock

classified into ground types A-B-C-D complying with the seismic code EN 1998-1:2004 are investigated. The soil profiles consist of saturated clays with different consistency, ranging from soft to rigid, resting upon a uniform linear visco-elastic bedrock. The mechanical parameters of the soil deposits and the underlying bedrock, intended as crisp or "best-estimate" values, are reported in Table 1; furthermore the table indicates the shear wave velocity of the soil calculated according to the relation  $V_s = \sqrt{\frac{G_0}{\rho}}$  with which each soil deposit has been characterized. The initially homogeneous profile is discretized in 0.5 m thick-layers assuming constant equivalent linear properties, compatible with the current shear strain computed at each iteration, according to the modulus reduction curve and the damping ratio curve proposed by Seed and Sun (1989) and Idriss (1990), respectively. The input seismic process applied at the outcrop bedrock is fully described by the stationary power spectral density,  $S_{\ddot{u}_0 \ddot{u}_0}(\omega)$ , determined from the response-spectrum-compatible model depicted in Figure 1, consistent with the soil type A and peak ground acceleration  $a_0 = 0.96 \text{ m s}^{-2}$ .

Table 1: Crisp soil parameters

Ground type	$V_s$ [m/s]	$G_0$ [MPa]	$\rho$ [kg/m <sup>3</sup> ]	$\nu$	$\xi_0$
A/bedrock	1000	$2.1 \times 10^6$	2100	0.45	0.025
B	400	$3.36 \times 10^5$	2100	0.45	0.05
C	250	$1.3 \times 10^5$	2100	0.45	0.05
D	150	$4.7 \times 10^4$	2100	0.45	0.05

#### 4.1 Fuzzyfication of the stochastic site response analysis

The fuzzy logic approach to the analysis entails a transformation process of the crisp or deterministic system parameters into fuzzy sets with grades of membership, referred to as fuzzification; different levels of fuzziness can be defined according to which parameters are considered fuzzy sets and which are not, being described by crisp or deterministic values. Afterwards, the hybrid fuzzy-stochastic site response analysis is conducted in order to provide fuzzy outputs. Our computational results are obtained by the Zadeh's extension principle, in combination with an implementation of the DE algorithm SPDE as described above. Analysis outcome is intended to provide information on both the quantification of the random and epistemic uncertainties on the main output results of the analysis and the propagation of the uncertainty through

the system; it is defined as follows:

$$\tilde{X}_{\tilde{U},i}(T_s, p, \zeta_i) = \eta_{U,i}(T_s, p, \zeta_i) \sqrt{\tilde{\lambda}_{0,\tilde{U},i}(\tilde{A})} \quad (12)$$

in which  $\tilde{\lambda}_{0,\tilde{U},i}(\tilde{A})$  is the fuzzy zero-order response spectral moment derived from Eq. (3) by considering fuzzy system parameters ( $\tilde{A}$ ), namely the soil/site properties involved in the site response analysis. In the present study, subjective informations are considered by constructing membership functions based on the knowledge acquisition procedure, preliminary carried out by several experts merging the objective available but imprecise information. The proposed input reference membership function has a symmetric trapezoidal shape described three characteristics: i) the core mid-point  $A$  of the parameter representing the "best-estimate" or crisp value, ii) the confidence, in non-probabilistic sense, or radius  $\sigma_{core} = A_1^{-,+}/A$  defined as the ratio between the edge of the core interval to the core mid-point, and iii) the confidence,  $(\sigma_{supp}^L = \sigma_{supp}^R) = A_0^{-,+}/A$  defined as the ratio between the edges left and right, respectively, of the interval of the support to the core mid-point.

The investigated uncertainties are the position of the bedrock that determines the soil deposit thickness  $h$ , the initial shear modulus  $G_0$ , the unit density,  $\rho$ , as well as the initial critical damping ratio  $\xi_0$  expressed as fuzzy sets, i.e.  $\tilde{h}$ ,  $\tilde{G}_0$ ,  $\tilde{\rho}$  and  $\tilde{\xi}_0$ , respectively. Based on subjective information, symmetric trapezoidal-shaped membership functions are constructed for describing the uncertain parameters for the four soil deposits considered in the analysis; Table 2 reports the parameters that determine each membership function. The fuzzy set  $\tilde{h}$  has  $\sigma_{core} = 0$ , thus it is a symmetric triangular fuzzy number whose core is crisp owing to the sharp interface boundary. It is worth noting that the classification of each soil deposit based on the crisp values, still remains valid for values on the core while for larger uncertainties, i.e. by decreasing the  $\alpha$ -cut, the same soil deposit might change rank.

The simulation is carried out by developing a numerical algorithm in MATLAB environment used to solve the  $\alpha$ -cut problem of at each  $\alpha$  level. Eleven  $\alpha$ -cuts are considered although five of them, namely  $\alpha = 0, 0.25, 0.5, 0.75$  and  $1$  are highlighted. Differential evolution method is applied in order to obtain the fuzzy extension of the function as defined in Eq. (12). Noteworthy, the result of the partial differential equation is the power spectral density thus a functional depending on frequency and fuzzy variables, therefore in order to deal with the optimization procedure, a parameter of synthesis is defined. In particular, in this paper, results of problem solving are presented in terms of fuzzy set of the median value  $\tilde{a}_{PGA}(\tilde{h}, \tilde{G}_0, \tilde{\rho}, \tilde{\xi}_0)$  of the largest peak of the acceleration at the top surface as follows:

$$\tilde{a}_{PGA}(\tilde{h}, \tilde{G}_0, \tilde{\rho}, \tilde{\xi}_0) = \tilde{X}_{\tilde{U}}(20, 0.5, 0) \quad (13)$$

in which  $T_s = 20s$ ,  $p = 0.5$  and  $\zeta_0 = 0$  have been assigned (see Eq. 3).

Table 2: Fuzzy sets parameters

Fuzzy set	Core mid-point	$\sigma_{core}$	$\sigma_{supp}^{L,R}$
$\tilde{h}$	40 m	0.00	0.2
$\tilde{G}_0$	$(210, 33.6, 13.0, 4.7) \times 10^4 \text{MPa}$	0.05	0.3
$\tilde{\rho}$	$2100 \text{kg m}^{-3}$	0.02	0.2
$\tilde{\xi}_0$	0.05/0.025	0.02	0.1

Results of the parametric analyses are described in terms of membership functions of the fuzzy output  $\tilde{a}_{PGA}$  for each of the four soil deposits. It is worth mentioning that each black dot on

the left and right side of the MFs is obtained by a specific quadruple  $(h, G_0, \xi_0, \rho_0)$  of crisp values belonging to the specific  $\alpha$ -cut of each fuzzy input calculated by solving the optimization problems (min and max) corresponding to the fuzzy extension of Eq. (13), i.e., for each  $\alpha \in [0, 1]$ , we have two quadruples of values  $h \in [\tilde{h}]_\alpha$ ,  $G_0 \in [\tilde{G}_0]_\alpha$ ,  $\rho_0 \in [\tilde{\rho}_0]_\alpha$ , and  $\xi_0 \in [\tilde{\xi}_0]_\alpha$ , corresponding to the minimization and the maximization problems.

## 4.2 Soil Type A

The fuzzy input memberships functions of the soil type A used in the analysis are depicted in Figure 2a while Figure 2b shows the analysis output in terms of membership function of the median peak ground surface acceleration. It is worth noting the the core mid-point value is exactly corresponding to the peak input ground acceleration  $a_0 = 0.96 \text{ m s}^{-2}$ , since the soil deposit is characterized by the same properties as the outcropping bedrock. The fuzzy output has a nonlinear LR-shaped membership function, fairly asymmetric with respect to the core towards the right branch as indicated by the dotted black curve collecting the mid-points of each interval associated with every  $\alpha$ -cuts calculated in the analysis. Therefore, a higher uncertainty on the input parameters, namely a small value of  $\alpha$ -cut, leads to an overestimate of the expected, possible, peak ground acceleration on the ground surface with respect to the mid-core value. In Table 3 are reported the parameters of the membership functions resulting from the analysis in terms of core mid-point  $a_{PGA}$  as well as core and support confidences  $\sigma_{core}$  and  $\sigma_{supp}^{L,R}$ , respectively. In soil type A, the measured global confidences of both the core and support of the result are lower than maximum values of confidences assumed for the describing the fuzzy uncertainties of the input soil parameters. Therefore, the reduced degree of the uncertainty indicates a small sensitivity of the seismic response for this type of ground.

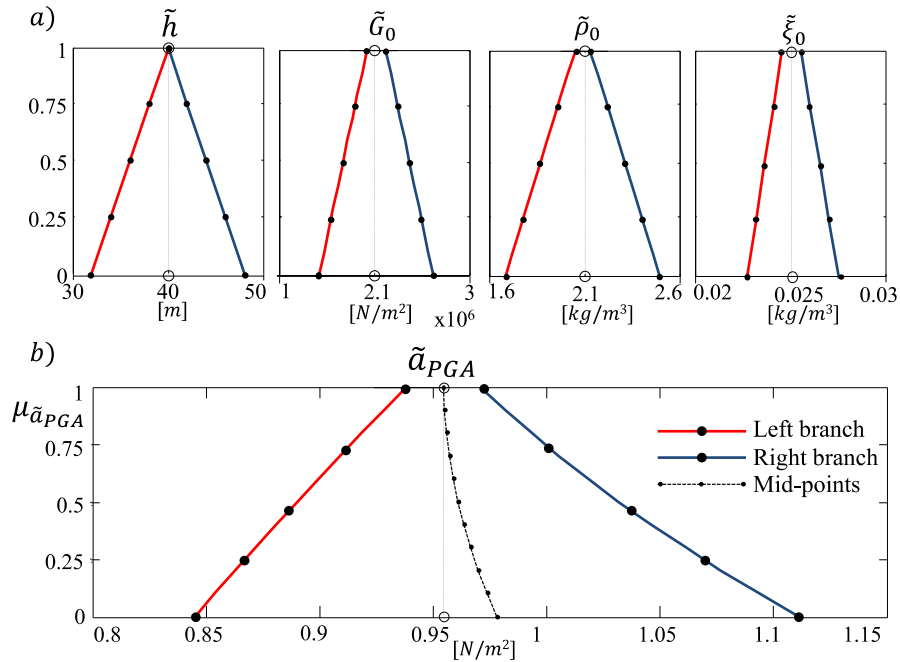


Figure 2: Membership functions of the: a) input soil parameters; b) surface peak ground acceleration for ground soil A

## 4.3 Soil Type B

Fuzzy input memberships functions for the investigated soil type B are depicted in Figure 3a. The result of the analysis in terms of membership function of the median peak ground surface

Table 3: Parameters of the output membership functions for the investigated soils.

Ground type	$a_{PGA}$	$\sigma_{core}$	$\sigma_{supp}^L$	$\sigma_{supp}^R$
A	0.960 $m/s^2$	0.017	-0.115	0.164
B	1.481 $m/s^2$	0.014	-0.121	0.164
C	1.483 $m/s^2$	0.011	-0.143	0.203
D	1.251 $m/s^2$	0.026	-0.297	0.282

acceleration is illustrated in Figure 3b. The fuzzy output has a nonlinear LR-shaped membership function, asymmetric with respect to the core towards the right branch as indicated by the dotted black curve resulting in overrating the expected peak ground acceleration on the ground surface with respect to the mid-core value. Moreover, as the previous case as shown Table 3, the uncertainty valued by the confidences is reduced with respect to the uncertainty assumed for the input parameters.

#### 4.4 Soil Type C

The investigated soil type C is characterized by the fuzzy soil properties depicted in Figure 4a. The result of the analysis in terms of membership function of the median peak ground surface acceleration is illustrated in Figure 4. The membership function of the fuzzy output is strongly nonlinear, asymmetric with respect to the core towards the right branch resulting in overrating the expected peak ground acceleration on the ground surface with respect to the mid-core value. It can be observed from the dotted black curve that the propagation of the uncertainty is associated with a change of the slope at around  $\alpha$ -cut = 0.5. Moreover, as shown Table 3, the maximum value of the confidence of the support is as high as the mean confidence assumed for the input parameters.

#### 4.5 Soil Type D

The result of the fuzzy optimization carried out by considering the fuzzy input memberships functions reported in Figure 5a for the last investigated soil type D is depicted in Figure 5b; the membership function of the median peak ground surface acceleration is characterized by fairly symmetric trapezoidal shape but converse to the previous case, the values of the mid-points are smaller than the mid-core value inducing an underestimate of the expected, possible, peak ground acceleration on the ground surface with respect to the crisp, deterministic value.

Furthermore, in soil type D, the measured global confidences of the support of the result are higher of the average confidence values assumed for describing the fuzzy uncertainties of the input soil parameters, reaching a value around 30% as shown in Table 3. Therefore, an important sensitivity of the seismic response is expected for this type of ground.

## 5 Conclusion

A fuzzy logic approach for dealing with soil uncertainties has been applied to the stochastic 1D site response analysis.

The fuzzy output is the median value of the largest peak of the accelerations at the ground surface determined for various  $\alpha$ -cuts representing the grade of membership to the set for 4 types of soil classified as A-B-C-D in accordance with the European Seismic Code. Results showed trapezoidal shaped membership functions, usually asymmetric towards the higher values except for the soil ground D. A strong influence of the soil uncertainties has been observed, in particular the effect of the nonlinearities becomes relevant on the propagation of uncertainty when for soft soil are considered.



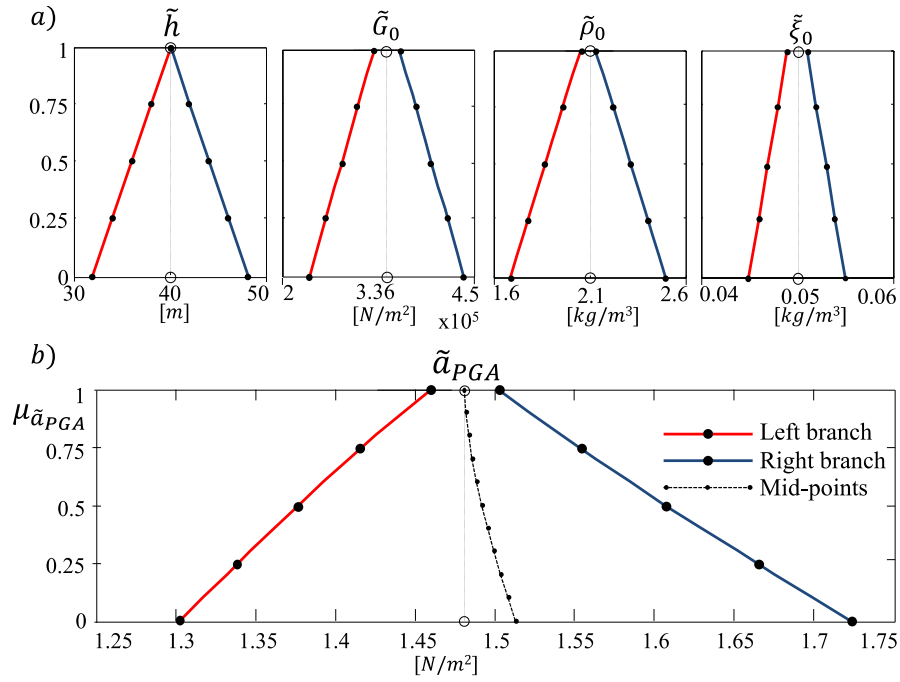


Figure 3: Membership functions of the: a) input soil parameters; b) surface peak ground acceleration for ground soil B

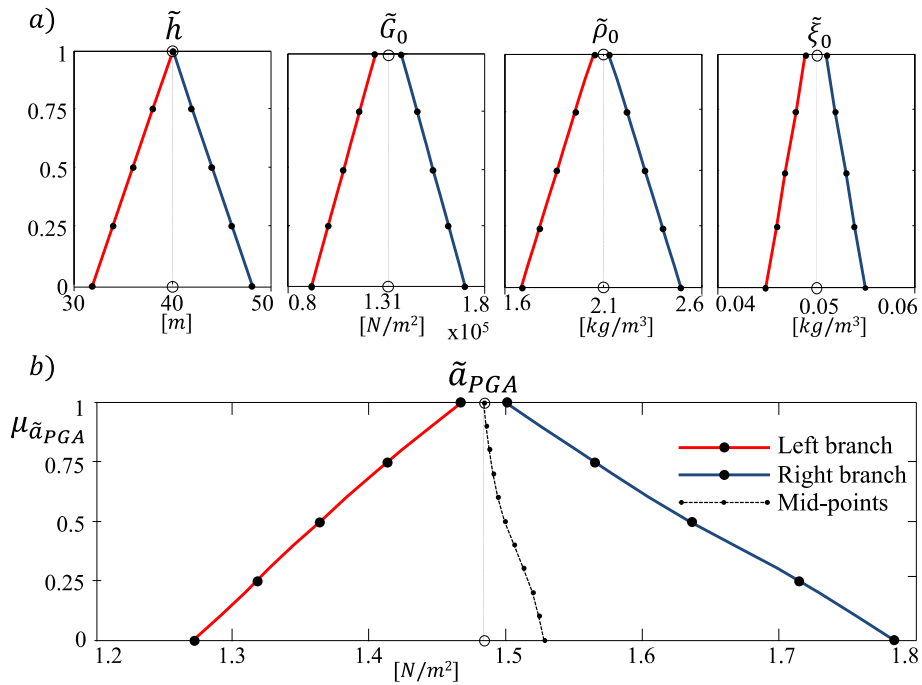


Figure 4: Membership functions of the: a) input soil parameters; b) surface peak ground acceleration for ground soil C

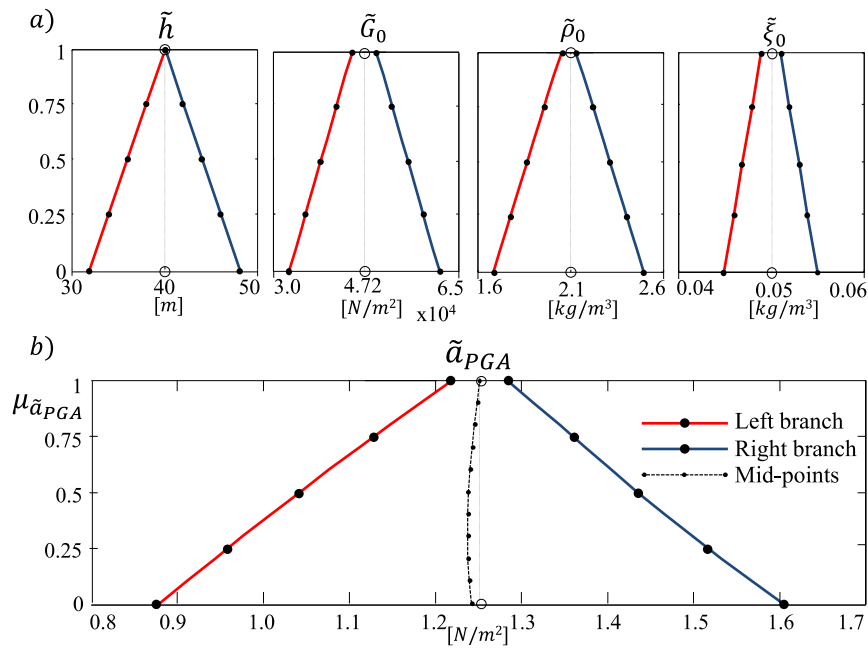


Figure 5: Membership functions of the: a) input soil parameters; b) surface peak ground acceleration for ground soil D

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