The Semiotics of Spider Diagrams

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Abstract. Spider diagrams are based on Euler and Venn/Peirce diagrams, forming a system which is as expressive as monadic first order logic with equality. Rather than being primarily intended for logicians, spider diagrams were developed at the end of the 1990s in the context of visual modelling and software specification. We examine the original goals of the designers, the ways in which the notation has evolved and its connection with the philosophical origins of the logical diagrams of Euler, Venn and Peirce on which spider diagrams are based. Using Peirce's concepts and classification of signs, we analyse the ways in which different sign types are exploited in the notation. Our hope is that this analysis may be of interest beyond those readers particularly interested in spider diagrams, and act as a case study in deconstructing a simple visual logic. Along the way, we discuss the need for a deeper semiotic engagement in visual modelling.

Mathematics Subject Classification (2010). Primary 03-02; Secondary 03A05.

Keywords. spider diagrams, logical diagrams, semiotics.

1. Introduction

The mid 1990s saw a remarkable growth of interest in logical diagrams which, after centuries of importance, had received less attention for the previous hundred years. Throughout the twentieth century, the success of symbolic logics appeared to reduce the diagrammatic notations of Venn, Peirce and their contemporaries to the status of novelties (see [31] for a survey of the history of diagrammatic logics and the processes that led to this period of relative neglect). Venn and Euler diagrams were valued, as they are today, for their ability to illustrate basic logical concepts to beginners, but not generally as a tool for logicians. The importance of Charles Peirce's graphical logics was recognised by some (notably Zeman [50], Roberts [35] and Sowa [40]) but the situation did not begin to change in earnest until the work of researchers such as Allwein [1], Barwise [4], Etchemendy [5] and others. The work of Sun-Joo Shin and Eric Hammer is of particular importance to this story:

separately [39, 18] and in collaboration with each other [19], Shin and Hammer were influential in reassessing the status of logical diagrams, analysing their formal properties and picking up again the task of understanding the origin of the potential advantages of diagrams over what had come to be seen by some as "real" logic. Their work at this time often included an *apologia* intended to rehabilitate the reputation of diagrams after their period of neglect in mathematics in general (e.g. [39, p3]).

Meanwhile, a plethora of software engineering diagrams had been developed since the 1970s. These were required in order to design, test and document conceptual models for software systems of ever increasing complexity. Notations included formal systems such as StateCharts [20], Entity Relationship Diagrams [9] and the semi-formal and unformalised notations that were combined to produce the Unified Modelling Language [26]. Approaches to the design of diagrams for software tended to be *ad hoc*; in 2002 it was still necessary for Hitchman to point out that "the details of conceptual modelling notations are important":

> Very little is documented about why particular graphical conventions are used. Texts generally state what a particular symbol means without giving any rationale for the choice of symbols or saying why the symbol chosen is to be preferred to those already available. The reasons for choosing graphical conventions are generally shrouded in mystery. [21]

Thus, we have two distinct and, for the most part but not entirely, disjoint communities and bodies of knowledge. In one, diagrammatic reasoning was explored in ways which carried on the projects of its originators. In the second, the goals of precise visual modelling answered the pressing needs of industry. This provides a brief and admittedly reductive sketch of the context in which *spider diagrams* appeared. Spider diagrams are based on Euler diagrams with the addition of syntax to represent individuals, and were first presented by Gil et al. in 1999 [15]. They arose within the visual modelling community but also took inspiration from the world of logical diagrams, especially in terms of their formalisation. Spider diagrams are notable for their influence within visual modelling and, of particular interest for this work, for their position at the meeting point of two diagrammatic cultures.

In the years since spider diagrams appeared Hitchman's advice has, to some degree at least, been heeded. The mathematicians, computer scientists, empiricists and others who study the application of logical diagrams in more or less scientific ways (whom we will call *diagram scientists*) call on results from cognitive science, human-computer interaction and elsewhere in order to understand existing notations and to design better ones. We believe that knowledge, terminology and approaches from semiotics and the philosophy of diagrams, however, are still underused, and that this has serious implications.

In this paper we have two main goals. Firstly, we will analyse, along Peircean lines, the notation of spider diagrams, identifying strengths and weaknesses of the diagrammatic language. Our hope is that this analysis may be of interest beyond those readers particularly interested in spider diagrams, and act as a case study in deconstructing a fairly simple visual notation. Secondly, we attempt to highlight and challenge a gap in terminology and technique between the philosophy of logic notation and the modern visual modelling community. In section 2 of this paper we describe the spider diagram notation and its evolution since its original appearance. In section 3 we analyse the semiotics of the notation with respect to Peirce's view of diagrammatic reasoning and sign function. We also revisit the goals of the designers of spider diagrams, making every effort to avoid the temptation of explanation via hindsight. We note the regular reinvention in diagram science of concepts familiar within the philosophy of diagrams. This leads us to conclude by examining what can be learned from a re-engagement with the ideas of those philosophers who produced, for very different purposes, the foundations of modern diagram science.

2. History and overview

Spider diagrams were first presented in two papers that appeared at the symposium *Visual Languages* 1999. These were *Formalizing Spider Diagrams* [15] and *Reasoning with Spider Diagrams* [24]. Since that time spider diagrams have had a significant impact within the visual modelling community, both in their own right and as the basis for more expressive logics. Part of the contribution they made in their own right was in the gradual refinement of an unambiguous abstract syntax suitable for developing meta-theoretical results such as logical completeness and expressiveness (for instance, [23]), and for the development and extension of techniques for reaching those results diagrammatically (i.e. without translating to symbolic forms known to be complete or to have a certain level of expressiveness) [8]. Research on the language was carried out mainly (but not only) at the Visual Modelling Group (VMG) at the University of Brighton, to which the authors belong. Other logics based on spider diagrams are *spider diagrams of order* [11], *constraint diagrams* [28, 43, 14], *generalized constraint diagrams* [41] and *concept diagrams* [42].

The first of this family of languages to appear was constraint diagrams, an example of which appears in figure 15 in the next section. Constraint diagrams are more expressive than spider diagrams, including as they do syntax for universal quantification and binary relations, and the full system is undecidable. In their earliest presentation, constraint diagrams suffer from ambiguity, since no reading order is imposed on the diagrammatic elements that represent quantifiers. Thus, spider diagrams were first produced by removing syntax from constraint diagrams in order to define a decidable fragment which would make the task of formalising the more expressive notation tractable.

In the rest of this section we will describe the spider diagram language informally, while also tracing some relevant aspects of the development of the notation. Full details of the formalisation can be found in [25].

2.1. Circles and existential spiders

Figure 1 shows a spider diagram, in which the boundary rectangle represents the universe of discourse. Closed curves represent sets and are labelled to indicate the set in question. Dots, or solid circles, are called *existential spiders*, or simply *spiders*, when there is no possibility of confusion with the other kinds of spider described later in the section. Spiders represent the existence of individuals in the sets denoted by the region in which they are placed. Spiders were inspired by Peirce's X-sequences, except that distinct spiders represent distinct individuals (other than in certain circumstances described below). In figure 1 the spider placed in the region inside A and B indicates that the set $(A \cap B) - C$ has at least one individual in it.

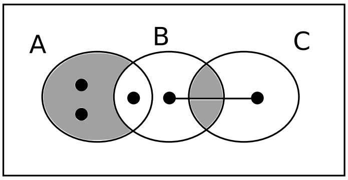

FIGURE 1. A spider diagram

Spiders can have several *feet*, meaning that several dots are joined by edges and denoting an individual in one of the regions in which its feet are placed. In figure 1 the spider with two feet indicates that something exists in either $B - (A \cup C)$ or $C - (A \cup B)$. Thus, at the concrete level, a spider is a graph and a spider with several feet provides disjunctive information. Labels on existential spiders are for convenience only; the individuals denoted are anonymous. The edges between a spider's feet are called its *legs* and the union of regions in which its feet are placed is called its *habitat*. An unshaded region with no spiders in it may or may not be populated. Shaded regions are empty apart from any individuals which are represented as spiders. Thus, the region in figure 1 that denotes $A - (B \cup C)$ contains at least two individuals, due to the spiders, and there at most two, due to the shading.

The diagram in figure 1 displays what we will call the *core syntax* of unitary spider diagrams: circles, shading and existential spiders. Furthermore, it is a *unitary* diagram; *compound* diagrams, representing the conjunction, disjunction and negation of diagrams are formed by the juxtaposition, joining by horizontal lines and placing horizontal lines above diagrams, respectively. The meaning of the diagram in figure 2 is equivalent to $(d_1 \wedge \neg d_2) \vee d_3$.

A piece of notation which was part of the original presentation but has not been pursued is the labelling of individual *zones* (regions in the diagram with no other region as a proper subset). Zone labels are distinguished from

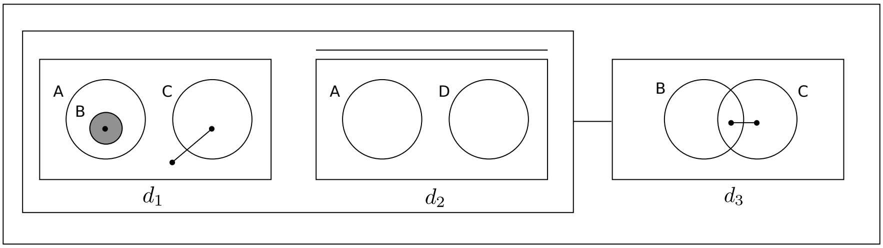

FIGURE 2. A compound spider diagram

contour labels by being underlined. Figure 3 shows the use of zone labels, where the zone labelled $\underline{D}$ shows the set $(A \cap B) - C$.

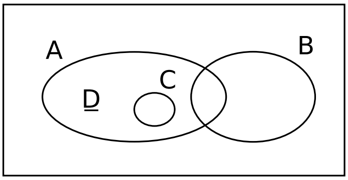

FIGURE 3. Zone labels

The original presentation also contained a number of other notational devices, all of which provide "syntactic sugar" but do not alter the expressiveness of the system (some of these devices do change the expressiveness of unitary diagrams). These are *projections*, which provide a way to show a particular set intersection of interest without needing to specify the entire context, and devices which alter the meaning of spiders: *ties*, *strands*, *constant spiders* and *Schrödinger spiders*.

2.2. Projections

A projection is a dashed contour which denotes the intersection of a set within a particular context. In figure 4, diagram (b) depicts the entire set A, whereas diagram (a) shows just that part of A which intersects with C and D. The projection in diagram (a) is labelled X but it requires an additional label, (A), which gives its *determining set*. The motivation behind this device was to produce less cluttered diagrams.

2.3. Other spiders

Strands and ties provide ways of joining distinct spiders to show that they *may* denote the same individual, or *do* denote the same individual, respectively. Strands are curved lines between spiders. In figure 5 the strand joining the two spiders in A but outside B asserts that they may represent the same individual. Strands increase the expressiveness of unitary diagrams since they can be used to depict a situation that would otherwise require several unitary

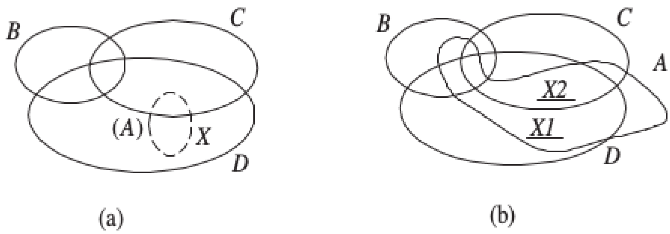

FIGURE 4. A projection and its semantics, taken from [15]

diagrams in disjunction. In same the same figure, B contains two spiders: one with two feet, placed in the zones denoting $A \cap B$ and $B - A$ respectively, and one with a single foot, placed in $B - A$. The two spiders are joined with a tie, or double line, for equality, asserting that they represent the same individual. Note that this means that the individual in question must inhabit a set denoted by a zone in which both spiders have a foot ($B - A$, in this case).

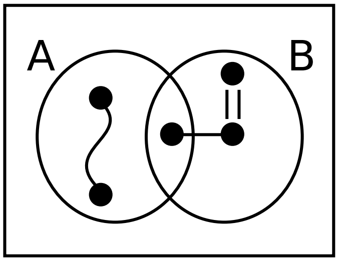

FIGURE 5. Strands and ties

Schrödinger spiders are small clear circles, or unfilled dots, and represent elements which might not exist. In figure 6 zone $\underline{C}$ is shaded and contains three Schrödinger spiders, meaning that it represents a set which is either empty or contains exactly one element, or exactly two elements, or exactly three elements. Note that if zone $\underline{C}$ were not shaded, the presence of these spiders would give us no information at all, since an unshaded region already contains *at least* as many elements as are depicted. This device was motivated by the common need in software modelling to depict optional elements.

Constant spiders are small solid squares that represent named individuals; figure 7 shows an example which says that an element named x exists and inhabits either $A - B$ or $B - A$. Like existential spiders and Schrödinger spiders, they may have several feet which may be joined by regular (straight) legs, strands or ties. Constant spiders do not extend the expressiveness of the logic [44] and are, again, inspired by the needs of software modelling.

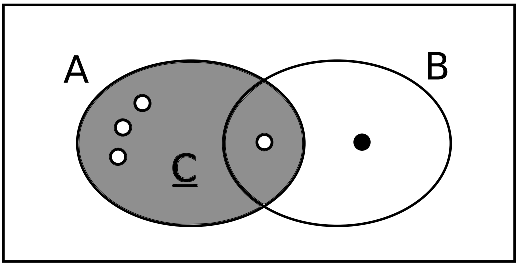

FIGURE 6. Schrödinger spiders

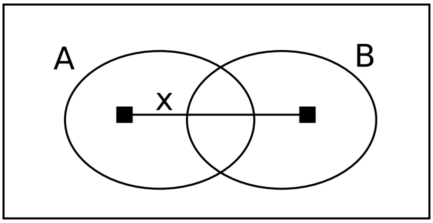

FIGURE 7. Constant spiders

2.4. Inference rules

The first paper on reasoning [24] defined the following five inference rules. This is not a logically complete set, and the motivation was to give a flavour of reasoning with the logic. A complete set of rules appears in [23].

2.4.1. Introducing a strand. This rule allows us to add a strand between two spider feet in the same zone or to replace a tie by a strand. In each case it weakens information. In figure 8 the diagram on the right can be obtained by two applications of this rule to the diagram on the left.

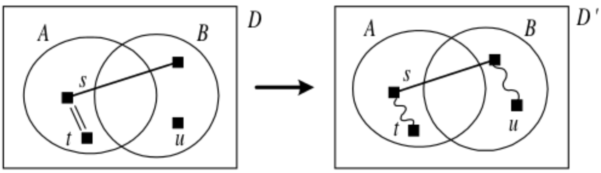

FIGURE 8. Introducing a strand

2.4.2. Erasure of an element. This rule allows the erasure of either a contour, the shading from a zone, an entire spider or the tie between two spiders. The original paper gives examples of how to do this while preserving the meaning of shading and spiders. See figure 9.

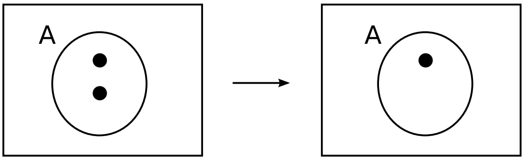

FIGURE 9. Erasing an element

2.4.3. Spreading the feet of a spider. This rule allows us to add a new foot to any spider, s, in a zone which is not currently part of the habitat of s. Figure 10 shows an example.

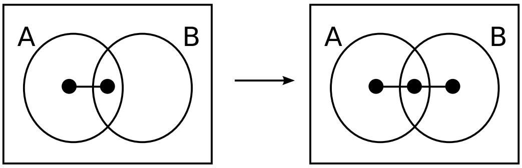

FIGURE 10. Spreading the feet of a spider

2.4.4. Introduction of a contour. We can add a new contour to a diagram so that it splits every existing zone into the parts which are inside and, respectively, outside of the new contour. Figure 11 demonstrates that this may mean adding feet to spiders to maintain validity.

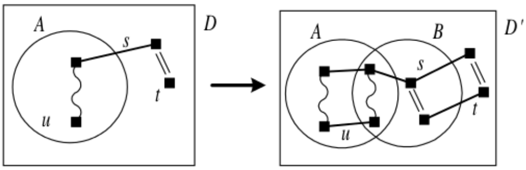

FIGURE 11. Introduction of a contour

2.4.5. Equivalence of Euler and Venn form. This rule allows us to replace a diagram in which some zones are missing with its *Venn form*, in which the zones which were previously missing are added as shaded zones. It also allows us to replace a diagram that contains shaded zones with one in which those zones are missing.

As we have seen, the original five rules were notably high level and can be used in different contexts to achieve different things. In later presentations the emphasis would be on simpler rules, which make it clearer which rules are being deployed in a proof and simplify the proofs of validity of the rules. In addition to rules above, the reasoning paper gave a process for combining two unitary diagrams, without formalising the process as an inference rule.

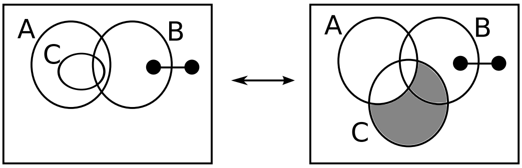

FIGURE 12. Equivalence of Euler and Venn form

In the example shown in figure 13, D_1 and D_2 are combined to produce D. The habitats of constant spiders s and t in D are the intersections of their habitats in D_1 and D_2. In zone $B - A$, s and t are connected by a strand in D_1 and a tie in D_2; hence in D they are connected by a tie. In zone $A \cap B$, s and t are separated in D_1 and connected by a strand in D_2; hence in D they are separated.

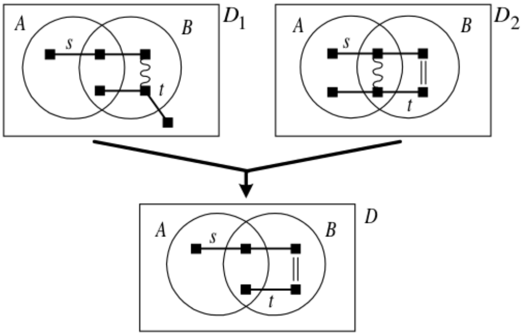

FIGURE 13. Combining diagrams, taken from [24]

2.5. Abstract syntax

In this paper we are mainly concerned with concrete syntax, as representing the "user interface" or final designed product of spider diagrams. But the usefulness of spider diagrams as a logic depended of course on formal properties such as the soundness of its inference rules, the completeness of a given set of rules, and so on. The task of cleanly separating details of drawn diagrams from their meaning was a major preoccupation, and all of these goals mean that the *abstract syntax* (or symbolic form) of spider diagrams is an important consideration, and so we give a brief flavour of it here.

In [15] an *abstract diagram* is defined as a tuple,

$$d = (\mathcal{C}, \mathcal{Z}, \mathcal{Z}^*, \mathcal{S}, \mathcal{S}^*, \eta),$$

as follows:

1. $\mathcal{C}$ is the set of contours in d.

2. $\mathcal{Z}$ and $\mathcal{Z}^*$ are the sets of zones and shaded zones, respectively (so $\mathcal{Z}^* \subseteq \mathcal{Z}$). A zone is given as a pair of sets of contours, (in, out), where in contains those contours inside the zone and out contains those contours outside the zone. It must be true that $in \cap out = \emptyset$ and $in \cup out = \mathcal{C}$, for every zone.
3. $\mathcal{S}$ and $\mathcal{S}^*$ are the sets of existential spiders and Schrödinger spiders, respectively, and $\mathcal{S}^* \subseteq \mathcal{S}$.
4. $\eta : \mathcal{S} \to \mathcal{R} \cup \emptyset$ is a function that gives the habitat of spiders.

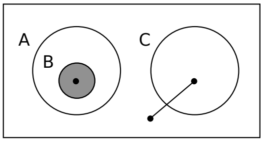

FIGURE 14. Abstract syntax

In figure 14, $\mathcal{C} = \{A, B, C\}$. $\mathcal{Z}$ is comprised of the following zones: $(\{A\}, \{B, C\})$, $(\{A, B\}, \{C\})$, $(\{C\}, \{A, B\})$, and $(\emptyset, \{A, B, C\})$. There is one shaded zone, and so $\mathcal{Z}^* = \{(\{A, B\}, \{C\})\}$. We omit the definitions of $\mathcal{S}$, for reasons of space.

2.6. Semantics

Following the style of Hammer and Shin, spider diagrams are given a model-theoretic semantics. A translation between elements of the abstract syntax and a given universe is established. A model is a triple (U, ψ, Ψ), where U is the universal domain, ψ maps spiders onto elements in U and Ψ maps contours onto subsets of U. The meaning of a spider diagram is then given as the conjunction of the following expressions:

1. The *Plane Tiling Condition*: the union of the sets depicted in the diagram is equal to the universal domain,
2. The *Spider Condition*: ψ maps spiders to elements of the sets represented by the zones those spiders inhabit,
3. The *Strangers Condition*: ψ maps distinct spiders to distinct elements of U,
4. The *Mating Condition*, spiders joined by ties really do represent the same individual, and
5. The *Shading Condition*: the set represented by a shaded zone contains no elements other than those represented by spiders.

This concludes the overview of the notation. In the next section we analyse the concrete syntax of spider diagrams and its relation to other logical diagrams.

3. Design and semiotics

We begin by examining the context in which spider diagrams were developed, including the goals of the designers, before analysing in turn each component of the notation.

First, did the designers' goals emphasise diagrammatic logic or visual modelling? The titles of the two original papers were *Formalizing Spider Diagrams* and *Reasoning with Spider Diagrams*. As the titles suggest, the initial emphasis was on spider diagrams as a logic, rather than on their use by non-logicians in visual modelling. Thus, the concerns of the authors were with such matters as creating adequate abstract syntax to guarantee the precision of the system, developing sound and complete sets of inference rules, establishing the formal expressiveness of the logic, and so on. Indeed, spider diagrams in themselves are not expressive enough for many software modelling tasks; as described in the previous section, they were first defined as a fragment of a more expressive notation which had been proposed as an augmentation to UML Class Diagrams [26], which would extend their expressiveness and precision. This motive positions constraint diagrams as straddling the disciplines of logic and visual modelling. As well as the syntax of circles, shading and spiders, constraint diagrams include *universal spiders* (asterisks) to represent universal quantification and *arrows* to represent binary relations. They allow the use of unlabelled (or *derived*) curves as the target (and, in some presentations, the source) of an arrow, allowing the user to determine in this way the domain and range of relations. Figure 15 shows an example. Several factors made the task of formalising constraint diagrams more difficult than anticipated. Firstly, note that the introduction of explicit universal quantification makes it necessary to impose a reading order on spiders. The diagram in figure 15 is intended to convey a constraint equivalent to the sentence "for all libraries, l, each copy of a book, c, in the collection of l relates to a particular publication, p, and each library reservation of c relates to the same publication, p". However, there is nothing to stop us reading one of the existential spiders first and coming up with a reading such as "there is a book which is in the collection of every library, and of which every copy is reserved".

Furthermore, the scope of universal spiders was problematic. In figure 15 the universal spider inside the curve labelled *Copies* inhabits a zone formed from the labelled curve *OnHold* and an unlabelled curve representing the image of the *collection* relation when its domain is restricted to *Libraries*. In this case, the universal spider in *Libraries* and the arrow sourced on it are required in order to construct the habitat of the universal spider in *Copies*. In figure 16, there are two universal spiders attached to arrows, each of which constructs the habitat of the other. This rules out a straightforward reading, unless we take the universal spiders as a quantifiers ranging over all of A and B. These questions relating to representing the structure of a formula within a constraint diagram were eventually solved, but they also prompted the focus on a more tractable and logically decidable fragment.

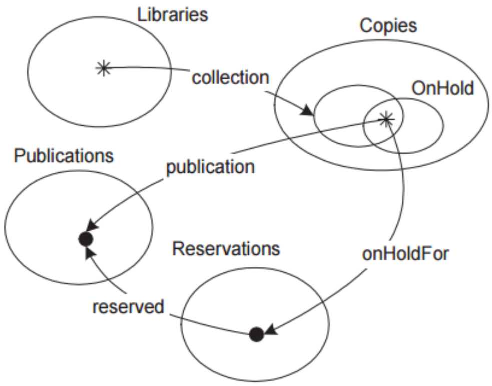

FIGURE 15. A constraint diagram, taken from [15]

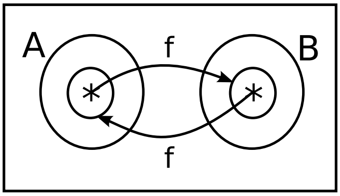

FIGURE 16. The scope of universal spiders

In 1994, when Shin's book on Venn diagrams [39] appeared, the task of formalising logical diagrams was a novel problem. The importance of distinguishing concrete from abstract syntax was not yet entirely understood, and although Shin was aware of this problem ([39, p49]), she does mix the two in places in that work; e.g. in section 3.3.1, *Set Assignments*, details of concrete diagrams are mapped directly onto the underlying semantics ([39, p64]). In joint work, Shin and several of the authors of spider diagrams were to solve a number of problems relating to the relationship between the semantic, abstract and concrete levels, a necessary task before notations such as constraint diagrams could be formalised effectively [22]. For these reasons, the complexity of constraint diagrams caused their designers to focus on a more manageable fragment, and this led to the development of spider diagrams. Despite the emphasis on diagrammatic reasoning in the earliest publications, the design goals could be said to have been inherited from constraint diagrams and reflect the concerns of visual modelling:

- give the user a lot of freedom about what they depict and how,
- devise features that produce less cluttered diagrams, and
- favour resemblance over convention.

We will revisit these design goals below when considering the elements of the notation. Next we examine the terminology and concepts used to reason about the effectiveness of spider diagrams and related logics.

3.1. Icons, well matchedness and free rides

Early publications on spider diagrams motivated the use of logical diagrams by appealing to their "natural" and "intuitive" properties (e.g. [15]), without attempting a precise explanation of what it might mean in practice to possess these properties. However this "naturalness" might arise, the argument was that it would enable people without logical training to make precise statements (specifically, to produce conceptual models of one sort or another). In more recent publications (e.g. [32], whose subject is concept diagrams, based on spider diagrams) the justification has become more systematic and proceeds along the following lines:

- Arguing that Euler-based notations make good use of Gestalt principles of grouping (e.g. similarity and continuity of form) [12].
- Appeals to phenomena such as Gurr's notion of *well matchedness* [17] and Shimojima's *free* (or *cheap*) *rides* [38, 37],
- Empirical work (e.g. Blake et al. [7]), interpreted with reference to the above.

Well matchedness and free rides are of special relevance to our discussion, both because they have been enormously influential ideas among diagram scientists and because, as we will explore, they are partial recreations of concepts first explained in detail by Peirce. Peirce's triad of semiotic modes (icons, indices and symbols) is also known and referred to within the modelling community but, arguably, its consequences and especially the nature of iconicity are not the subject of prolonged discussion.

We begin by examining well matchedness. In his 1999 paper [17], Gurr proposes the term and associates well matchedness with *intuition* and *naturalness* (possibly giving the grateful diagram scientist a technical term to replace two which are a little doubtful and vague):

> Taking seriously the assertion that diagrams are useful because they are intuitive (well matched to meaning) and natural (directly capture this well matched meaning), then for any formal underpinning of diagrams in software engineering to be truly useful it must therefore reflect these intuitive and natural aspects of diagrams. [17]

Gurr uses Euler's circles to represent the *BARBARA* syllogism and notes that the diagrams provide "a classic example of [the] direct capture of semantic information in a visual symbol [...] The transitive, irreflexive and asymmetric relation of set inclusion is expressed via the similarly transitive, irreflexive and asymmetric visual of proper spatial inclusion in the plane." That is, Euler's circles are *well matched to their meaning.* Gurr shows that representing the same problem with Venn diagrams results in a less well matched representation, indicating that this is an analogue property which yields a scale or spectrum. This is generalised as follows, in which the appeal to resemblance appears to mark the rediscovery of (part of) Peircean iconicity:

> [T]he effectiveness of a representation is to a significant extent determined by how closely the semantics of the representation resembles that which it represents. One benefit that certain diagrammatic representations offer to support this is the potential to directly capture pertinent aspects of the represented artifact (whether this be a concrete artifact or some abstract concept). [17]

How does the language of diagrams science correspond with Peirce's semiotics, in particular with a nuanced notion of iconicity? Well matchedness as correspondence between syntax and semantics is the essence of Peirce's earlier definitions of iconicity: an icon is "a sign that functions by means of shared characteristics between the sign and the object" [46]. Resemblance is a notoriously slippery concept, and Stjernfelt notes the questions surrounding this definition: "what is the basis of the 'similarity' between the sign and its object? Is it a psychological experience of resemblance, or a relational property?" [47]. In the case of well matchedness, at least, Gurr intends a relational property or isomorphism. But the essence of iconicity and, moreover, of logical diagrams themselves as far as Peirce is concerned, is the capacity for "learning more": an icon is a sign "from which more information may be derived" (CP 2.309, quoted in [46]). That is, the iconic sign and its object are reliably similar under certain transformations. The icon is "the only sign by the contemplation of which more can be learned than lies in the construction of the sign" [47]. Writing in 1996, Shimojima describes the phenomenon of *free rides* as "[one of the] ways in which operational constraints over diagrams intervene in the process of reasoning" [37]. Free rides occur when the topological properties of a diagram force some logical consequence to be made explicit, where that consequence would otherwise take several inferential steps. Diagrams that exhibit free rides are certainly signs from which we can "learn more". Gurr, once again: "certain inferences are somehow more immediate, or even automatic, in diagrams." In fact, Gurr appears to absorb the idea of free rides into well matchedness:

> [A]n intuitive, or well matched, representation is one which clearly captures the key features of the represented artifact and furthermore simplifies various desired reasoning tasks. [17]

Both well matchedness and free rides have been influential in visual modelling and among diagram scientists. For example, according to Google Scholar, Gurr's paper *Effective diagrammatic communication* [16] (which introduced well matchedness) has 120 citations at the time of writing, a large number for the modestly-sized field; Shimojima's paper *Inferential and expressive capacities of graphical representations: Survey and some generalizations* [38], which introduced free rides, has 76. In fact, we claim that both well matchedness and free rides are ideas that have become internalised within diagram science and, as such, are frequently mentioned or implicitly assumed without there necessarily being a perceived need for a citation. The two notions – a resemblance relation and the direct exposure of logical consequence – reflect different aspects of iconicity. Taken together, they match Stjernfelt's notion of *operational iconicity*: the necessary condition of an iconic sign. Operational iconicity is not the last word on this sign property however, since Peirce also described icons as signs which describe things "as they really are". Icons which reach this high standard of fidelity to reality are called *optimally iconic* by Stjernfelt [47], an idea which we explore below. In the rest of this section we use these ideas in our attempt to analyse the component parts of spider diagrams and their interaction.

3.2. Circles and shading

The use of circles in spider diagrams mixes the semantics of Euler's circles [13] and the later extensions of Venn [49] and Peirce (CP 4.360 [33]). Disregarding spiders for the moment, the presence of a region in the diagram does not imply that underlying set is inhabited and shading represents emptiness (Venn-type semantics). Where curves in the diagram do not overlap, this gives rise to a "missing region" which is known to be empty (Euler-type semantics). Regarding these choices, Gil et al. wrote: "In view of their relative merits [i.e. Euler's circles and Venn/Peirce diagrams], it seems natural to combine the two notations." [15] The absence of existential import allows the representation of partial information, something which is essential in the context for which spider diagrams were designed. Crucially, it avoids the over-specificity (noted by Venn and Peirce, and which Shimojima calls *overdetermined alternatives* [37]) of Euler's circles by allowing the depiction of things which may not exist. This flexibility is achieved by sacrificing some of the prior commitment to reality. The interaction between spiders and shading departs from X-sequences in Venn-Peirce diagrams, in which a shaded region is certainly empty, meaning that a shaded region that contains an X-sequence is inconsistent. Spider diagrams relax this rule so that a shaded region can contain spiders. Rather than denoting emptiness then, shading provides an upper bound on cardinality, while spiders provide a lower bound, enabling spider diagrams to assert precise cardinalities. Because the set denoted by an unshaded zone either contains exactly as many elements as are depicted by spiders, or at least one more, we can define a rule that carries out the inference shown in figure 17. The diagram to the right of the turnstile is a disjunction of two unitary diagrams, as explained below.

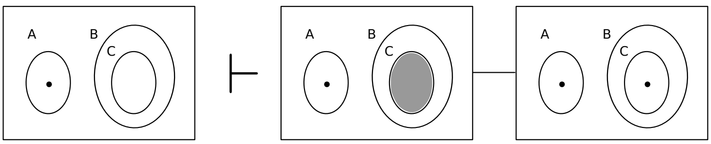

FIGURE 17. C is either empty or has at least one thing in it

The direct precedent for this use of circles is from Hammer and Shin's version of Euler diagrams (see *Euler's Visual Logic* [19]). Hammer and Shin use the terms *class* and *set* interchangeably (though they are referring to sets), and there appears to be no important distinction for them between sets and the term Euler used, *notion générale*. In one sense, it may be that pragmatism and the goal of flexibility of expression trump a certain type of philosophical verisimilitude for the authors of modern logical diagrams. The choice to focus on classes or sets, and whether it is important to make a distinction between them, is one way of choosing whether it is coherent to depict things which may not exist. But this is part of a wider pattern which Moktefi and Shin describe in their survey [31], whereby the "needs and requirements of the logicians who invented the diagrams" continue to change over time and are reflected in the systems they construct.

In recent work Moktefi explores the semiotics of Euler's circles [30] and concludes that they are not only iconic, but optimally so. This is demonstrated using an experiment which presents a spatial set (or proposition) visualisation similar to Euler's circles but based on intension rather than extension: "In a diagrammatic scheme founded on intension, a term A (respectively B) is represented with a circle that gathers all the attributes that are predicated of A (respectively B). Hence, a proposition asserting that 'Every A is B' would be represented by a circle B inside a circle A." [30] Using the intensional notation to reason with syllogisms is something like stepping through the looking glass. Containment of circles no longer means subsumption of sets, but just the opposite. For the same type of syllogistic task at which Euler's circles excel, the intensional notation hinders rather than helps reasoning. This topsy-turvy notation *is*, however, well matched to meaning, in that we can establish an isomorphism between diagram regions and the semantic level. Although the diagrams do not exhibit free rides, we could manipulate these intensional diagrams in reliable, albeit constrained, ways once we understand how the notation works. This experimental notation demonstrates the limitations of Peirce's resemblance-based definitions of icons and of the "structural correspondence" reading of well matchedness; circles *do* resemble intents, for some definition of resemblance, and so the diagrams *are* well matched but have no convenience of use for reasoning. It appears that operational iconicity is a necessary but not sufficient reason for free rides.

Moktefi contrasts this with the case of Euler's circles which "*have* the relations that they are said to represent", giving them the capacity to represent classes *as they really are*. This capacity depends on the existential import of regions – any region enclosed by a curve represents a class with something in it, and so only things which exist are depicted. The claim that this can allow intuition to support inference is supported by an empirical study by Sato et al., which found that readers make significantly fewer mistakes with Euler's circles than with Venn diagrams [36].

Thus, while diagram scientists partially reconstructed the notion of iconicity in the closely related concepts of well matchedness and free rides, some concepts and terminology are still lacking if we are to describe the way signs function within spider diagrams; specifically, the notion of *optimal iconicity.* As already noted, the use of circles in spider diagrams does not reach that standard, due to shading and the absence of existential import. Circles represented "general notions" in Euler's original presentation. We presume a general notion is something which must exist in order to be discussed or depicted. In this context, it makes perfect sense that the way to depict the non-existence of a notion is leave the relevant region out of the diagram altogether. Figure 18 illustrates this, in which zones such as that containing A and C but not B or D are missing. Figure 19 demonstrates the more general approach of Venn – what were non-existent classes in Euler's circles are now empty sets, as indicated by shading. A spider diagram with the same meaning is shown in 20, demonstrating a still more general approach that would admit either of the two preceding styles.

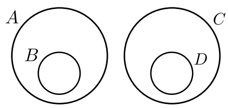

FIGURE 18. Missing zones in Euler's circles

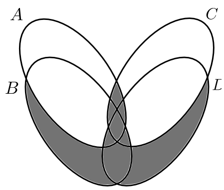

FIGURE 19. "Missing" zones in a Venn diagram

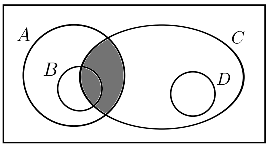

FIGURE 20. "Missing" zones in a spider diagram

To summarise, shading and the removal of existential import act to reduce the iconicity of circles in spider diagrams. The retreat to a more symbolic or conventional mode brings much flexibility of expression. This pattern is repeated in other syntactic choices – however powerful iconicity may be, especially with regard to making explicit the consequences of a logical expression, it is at times *too* powerful. A commitment to saying things "as they really are" means removing choice about *how* they are said, resulting in overdetermination. With symbolic features, we choose the convention to adopt, tempering the doctrinaire power of resemblance. That freedom to choose between styles of expression is of particular importance for the visual modelling task, given the prevalence of partial information. Nevertheless, when compared to Euler's circles, with all their limitations, the innovations are in some sense an anti-diagrammatic turn. The new flexibility is influenced in this case at least partly by software engineering, as much as Venn's innovations were influenced by his interest in symbolic, especially Boolean, logic [31]. As Venn diagrams mark the move away from syllogistic reasoning to the increasing complexity of symbolic logic, so the use of circles in spider diagrams mark the new use case for logical diagrams of generalised conceptual modelling.

3.3. Projections

Many logicians, including Peirce, previously used the dashed line to suggest *possible* information, whereas projections represent information which is certain but partially specified. The diagram in figure 21 deploys one of its circles in a symbolic and conventionalised way, requiring a kind of "higher order" of visualisation on the part of the reader, who may only speculate about where else the set A appears after the parts of no interest are shorn away.

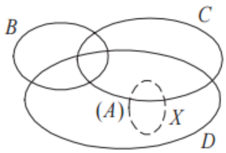

FIGURE 21. Projections and higher order visualisation

Projections are primarily a powerful way of getting rid of *clutter*, or redundant syntax [27], as demonstrated by the example in section 2, figure 4. Clutter has been shown to have important practical implications for creating diagrams people can readily understand [2]. The dashed curve of a projection introduces a symbolic dimension to the usual meaning of a closed curve drawn with a solid line. There are important differences in the ways each type of curve denotes a set. For instance, it is not valid to remove an unshaded zone from a unitary diagram but it is always valid to "reduce" a projection, since that merely means that a smaller intersection is to be displayed. Thus, projections are a device that diminishes iconicity for practical reasons. This is no small matter, since it reduces the capacity of the diagram to offer free rides and to support intuition; one might say it reduces the capacity of the diagram to support *inference*, in fact. It is done, however, to enable the modelling of complex ideas in succinct ways. As an example of the distance travelled from the satisfyingly simple, iconic properties of earlier spatial logics, figure 22 shows a Venn-6 spider diagram constructed by embedding one Venn diagram inside another. (We abuse the notation here to make a more legible diagram; each projection should really have its own label.) Interestingly, and although the spider diagram authors were not aware of it at the time, projections appear to have been anticipated in a diagrammatic experiment by Peirce; Pietarinen examined Peirce's manuscript example of embedding Venn-4 in Venn-4 in [34].

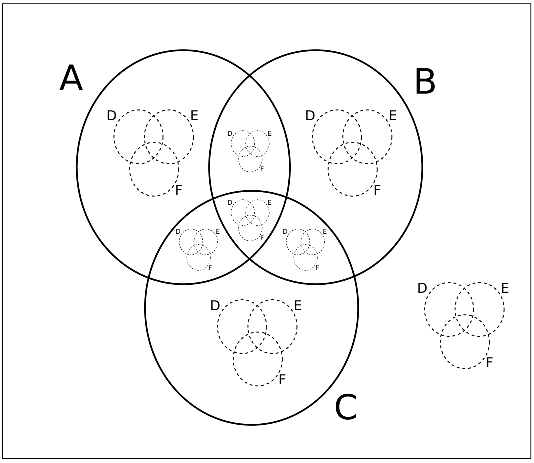

FIGURE 22. Venn-6 with projections

3.4. Spiders

Spiders, the most distinctive and eponymous feature of this notation, express lower bounds on cardinality – any region contains at least as many elements as there are spiders entirely contained within that region. Used in combination with shading, which places an upper bound, spiders allow the assertion of exact cardinalities, and spider diagrams were the first widely known logical diagram to do so.[1]

Just as Peirce believed the ways in which icons, indices and symbols are perceived correspond to and may provide examples of his three phenomenological categories (*firstness*, *secondness* and *thirdness*) [45], so he also believed that icons, indices and symbols should be used to depict the types of information that pertain to the corresponding categories. These are *hypotheses*, *facts about experience* and *general laws*, respectively ([33, 3.460] cited in [29]). Thus, predominantly indexical signs should be used to denote facts about experience, such as existence, and this matches the purpose and function of spiders well. An existential spider with a single foot is strongly indexical, asserting that "there is something here", something that falls into the category of secondness, exemplified by those things we may "put our shoulder against" ([33, 3.240], cited in [3]). The single-footed spider also has iconic properties arrived at via its placement in the diagram. A diagram in which each spider has a single foot, such as that in figure 23, is called an α-diagram.

A purely conventional feature (the "leg", or edge between feet) is used to create spiders with more than one foot. The spider's leg denotes disjunction, but could just as well mean something else. (We don't have much choice about whether to use a conventional feature – Shin noted the difficulty of depicting

[1] Peirce appears to have achieved this first too, as shown in marginalia uncovered by Pietarinen [34].

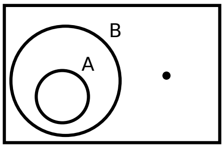

FIGURE 23. An α-diagram

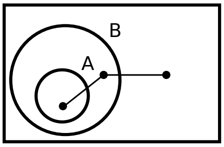

FIGURE 24. Spiders with several feet

logical disjunction using resemblance [39, p180].) The indexical property of the spider is thus diluted: "there is something here, or here, or here..." (see figure 24). Spiders with several feet then provide an example of a sign that functions in all three modes. Firstly, the feet themselves are indexical. You can "put your shoulder" against them, in Peirce's memorable phrase. The placement of the feet is iconic since their presence in a certain part of the diagram "has the relation it depicts", and free rides may occur because of the assertion made by placement, allowing us to learn more. Finally, the legs are symbolic, being a convention for disjunction.

Spiders' legs increase the expressiveness of unitary diagrams, but not of the system as a whole. We could do without them by forming a disjunction of unitary parts, as in figure 25, which has an equivalent meaning to figure 24. Again, a conventional, less diagrammatic addition (legs) allows for succinct expression, getting rid of clutter.

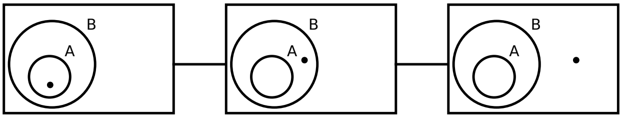

FIGURE 25. A disjunctive spider diagram

Constant spiders do not increase expressiveness (see [44] for a formal discussion) but are very useful for modelling.[2]. Strands, ties and Schrödinger spiders fall into the category of conventional devices which reduce clutter. Strands and ties diminish the iconicity of spiders, since a single spider no longer necessarily represents a single individual.

Alternative semantics for ties are possible. We might decide that the two elements are equal *only if* they inhabit the same set, i.e. that the tie denotes a relationship between spiders' feet, rather than the spiders themselves. The semantics given in the previous section (spiders represent equal elements whenever they are joined by a tie) gives rise to the only form of inconsistency in a unitary diagram, in fact. Figure 26 demonstrates the problem, as there is no model in which the spider with three feet could be equal to both of the single-footed spiders.

[2]Though focusing on Venn rather than Euler diagrams, Choudhury and Chakraborty were also to explore the use of named individuals in their 2004 work [10]

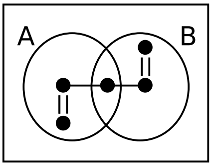

Figure 26. Inconsistency from ties

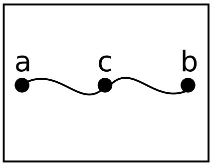

Figure 27. Transitivity and strands

A similar transitivity issue applies to strands. In figure 27 we may want to assert that a and b may be equal, and b and c may be equal, but a is definitely not equal to c. This non-transitive interpretation, whereby both strands and ties govern only the particular foot to which they are attached, leads to more explicit diagrams having less potential for misunderstanding but with a corresponding increase in clutter.

The Schrödinger spider represents an optional value, sometimes called a "Maybe" type, something which plays an important part in software specification. In practice, however, this spider is impossible to tell apart from a small curve. As well as the unfortunate possibility of confusing two types of notation which are meant to convey different things, Peirce's pairing of sign types with types of information described above may give us a clue as to why Schrödinger spiders may be problematic. Here, a sign (the circle) which is elsewhere used successfully to depict "theorems" in an iconic way (for instance depicting the theorem $A \subseteq B$ by placing one circle inside another), is used to denote the possible existence of an individual, a denotation which calls for an indexical sign.

3.5. Compound spider diagrams

Finally, we consider the various ways in which unitary diagrams can be joined to form compound diagrams. Compound diagrams express the conjunction, disjunction or negation of their unitary components. The first presentation used a minimal approach: juxtaposition for conjunction and unitary diagrams joined with horizontal lines to represent disjunction. Both notational choices were used by Peirce but entered spider diagrams through Shin's Venn-II. A horizontal bar above a unitary diagram represents negation. Nested boxes provide bracketing, or scope. See figure 28 for an example.

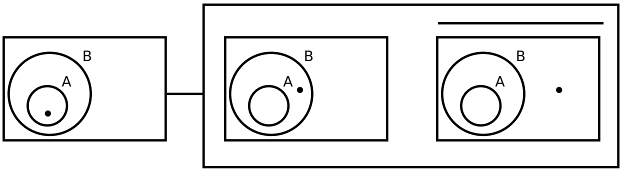

Figure 28. The original form of compound diagrams

Like projections, diagrams featuring several boundary rectangles require another piece of "higher order" visualisation from the reader. If the rectangle represents the universe of discourse then what does it mean to depict several of them at once? It may be straightforward to see that two such universes at the same level (i.e. without any nesting) are meant to be read in conjunction (as two aspects of a single universe) or disjunction (as two possible universes). When nesting one boundary rectangle within another these abstractions can obviously begin to mount up, but there is probably no possibility of a "natural" depiction of such complex and abstract ideas.

Later versions of the notation experiment with a more elaborate scheme whereby unitary diagrams are enclosed in a rectangle which has a conventional symbolic operator written in the top left corner. The diagram in figure 29 has the same meaning as that in figure 28.

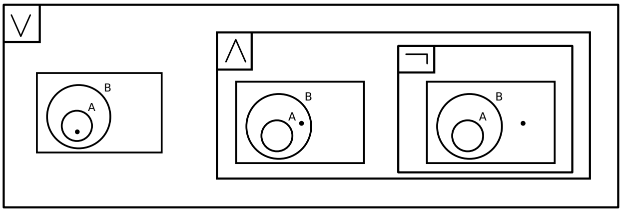

Figure 29. The later form of compound diagrams

This second approach was not pursued. We would argue it was less successful than the original form of compound diagrams for two main reasons; it makes poor use of space and is less diagrammatic. Juxtaposition clearly seems to be an iconic and effective way to represent conjunction, a diagrammatic form which has the relation it represents; the existence or presence of the two things juxtaposed. In the style of figure 29, all unitary parts within a bounding rectangle are juxtaposed and one must read the mathematical symbol to see which logical relation is in effect. Last but not least, the earlier way of forming compound diagrams emerges from a long-established tradition familiar to many readers and, as Peirce observed in his remarks on the ethics of notation, we should acknowledge and learn from previous notational efforts; if we go on to reject them, we should at least have a good reason for doing so (MS 530, 1904, see footnote in [6]).

3.6. Inference rules

The rules in [24] are given informally (i.e. without using the abstract syntax). An overall goal in designing the rules was that they should be diagrammatic. That is, they should be inspired by the forms of diagrams rather than being translations of algebraic identities into diagrammatic form. The idea was not, in the original presentation, to present a minimal or logically complete set of rules, or for the rules to be atomically simple, but to give a flavour of the notation and of the forms reasoning with it would take. The result, however, is that the rules are defined in ways which are unnecessarily, if deliberately, broad. Rule 2, Erasure of a diagrammatic element, is one such rather unfocused rule; the elements that can be removed from diagrams are shading, spiders and circles. If formalisation was the primary concern it would have been preferable to define simpler rules, one for each diagrammatic element. More importantly, simpler rules would benefit the users of the logic when choosing which rule to apply or following a proof. Similarly, rule 5, Equivalence of Venn and Euler forms, is a high level rule that does several things at once. The same purpose, converting a diagram into Venn form, could have been better served by a simpler rule which allows adding a single shaded zone, which could then be applied repeatedly.

Several of the rules do have an appealing visual perspicuity: if the reader knows that spider's leg represent disjunction then the example given of Rule 3, spreading spiders legs, should need no further explanation. There are several ways in which we could define Rule 4, which introduces a contour. For instance, there is no need to depict the intersection of the new contour with regions that are known to be empty. However, the rule is defined in an, arguably, more intuitive way by splitting every existing region in two.

Rules 1 and 3, concerning strands and spiders' feet respectively, are weakening rules. There was a tendency in later presentations to void weakening rules, defining inferences instead as equivalences for the convenience of the designers (e.g. in order to aid the process of proving completeness).

We have shown in this section that the spider diagram notation extends and adapts systems such as Venn-II. We have tried to trace the way in which many, but not all, of these changes were made primarily, at least in the earliest presentations, with conceptual modelling rather than logical reasoning in mind. In summary, the new devices such as projections add flexibility of expression whilst frequently diverging from the goal of creating as direct a reflection as possible between meaning and the drawn diagram.

4. Conclusion

Echoing the title of a well known work on the philosophy of science, a possible subtitle for this article could be *The Spontaneous Philosophy of Spider Diagrams*. We have observed that scientists have a tendency to underestimate or ignore the impact of implicit philosophies in their work, or to believe that their only guiding principles are pragmatism and the "scientific method". Unless the philosophies which drive the work of diagram scientists are made explicit then they cannot be reflected on and are effectively impossible to modify, giving them the status of ideologies. The self-directed accusation is that diagram science needs to be reconnected to the concerns and goals of the philosophers that produced the logical diagrams on which their work is based.

In the previous section we presented several examples of the robustly practical principles which drove the design of spider diagrams. We have also noted that this is part of the natural evolution of logical diagrams, as they are deployed to represent more and more complex and abstract states of affairs. As a final example, Gil et al. noted that we can't use spider diagrams to say $|A| = |B|$ for disjoint sets A and B: "In order to retain the intuitiveness of spider diagrams, constraints such as this should be placed in an *auxiliary textual annotation*" [15] (emphasis added). Does this readiness to abandon not only faithfulness to reality but also diagrammatic form itself in order for convenience, flexibility and expressiveness matter? In terms of the fitness-for-purpose of spider diagrams, almost certainly not. But these questions did of course matter to the authors of the original notations on which spider diagrams are based. Icons, indices and symbols are not only linked by Peirce to his three categories of experience, but are exemplars of the categories [45]. In this way, a diagram made of icons, indices and symbols forms a "categorical mirror" that reflects nothing less than human experience. This ambitious diagrammatic vision is bold but should not be surprising as it merely places logical diagrams on a level with philosophical logic more broadly as a tool to analyse and explore reasoning and knowledge acquisition. This is beyond the scope of the current work, but is mentioned to highlight the philosophical commitment behind the design of Peirce's sign triad and his semiotic in general. The designers of modern logical diagrams could be said to have shown an equally firm commitment to pragmatism, but would do well to bear in mind that the component parts of the diagrams they mould into new forms were not chosen at random.

A better understanding of semiotics and the principles underlying other logical diagrams can help those people who "only" want to use them in practical ways (rather than considering their place in any deeper, e.g. phenomenological, framework). In fact, we have so far made use only of Peirce's earliest, least extensive (and most tractable) sign typology. We are interested to see what can be learned by analysing modern logical diagrams from the perspective of the more detailed mid and late sign typologies [48][3, p150]. Despite the possible limitations of our analysis, we uncovered several examples of semiotic concepts that were needed, and thus reinvented, in diagrams science. Spider diagrams were arrived at by extending Euler diagrams in a more or less purely pragmatic way, and the result proved to be a useful development on the road to more expressive notations. But diagrams science needs to be reconnected with its origins, and in particular with a nuanced understanding of the power and drawbacks of iconicity and resemblance.

4.1. Acknowledgements

Thanks to past and present members of the VMG and other collaborators and friends who worked on spider diagrams. Thanks also to Francesco Bellucci for helpful comments on a draft of this article that led us to clarify the discussion.

References

[1] G. Allwein and J. Barwise, editors. *Logical Reasoning with Diagrams (Studies in Logic and Computation)*. Oxford University Press, 1 edition, June 1996.

[2] M. Alqadah, G. Stapleton, J. Howse, and P. Chapman. Evaluating the Impact of Clutter in Euler Diagrams. In T. Dwyer, H. Purchase, and A. Delaney, editors, *Diagrammatic Representation and Inference*, volume 8578 of *Lecture Notes in Computer Science*, pages 108–122. Springer, 2014.

[3] A. Atkin. *Peirce (The Routledge Philosophers)*. Routledge, July 2015.

[4] J. Barwise and J. Etchemendy. *Heterogeneous Logic*, pages 179–200. Oxford University Press, Inc., NY, USA, 1996.

[5] J. Barwise and J. Etchemendy. *Language Proof and Logic*. CSLI, 1999.

[6] F. Bellucci and A. V. Pietarinen. Existential Graphs as an Instrument of Logical Analysis: Part i. Alpha. *The Review of Symbolic Logic*, FirstView:1–29, February 2016.

[7] A. Blake, G. Stapleton, P. Rodgers, and J. Howse. The impact of topological and graphical choices on the perception of euler diagrams. *Information Sciences*, 330:455 – 482, 2016. SI Visual Info Communication.

[8] J. Burton, G. Stapleton, and J. Howse. Completeness Strategies for Visual Logics. In *7th International Conference on the Theory and Application of Diagrams*, 2012.

[9] P. P. Chen. The Entity-Relationship Model – Toward a Unified View of Data. *ACM Trans. Database Syst.*, 1(1):9–36, March 1976.

[10] L. Choudhury and M. K. Chakraborty. On extending Venn diagrams by augmenting names of individuals. In *Proceedings of 3rd International Conference on the Theory and Application of Diagrams*, volume 2980 of *LNAI*, pages 142–146. Springer-Verlag, March 2004.

[11] A. Delaney, J. Taylor, and S. Thompson. Spider Diagrams of Order and a Hierarchy of Star-Free Regular Languages. In *Spider Diagrams of Order and a Hierarchy of Star-Free Regular Languages*, pages 172–187. Springer, 2008.

[12] J. Enns. Gestalt Principles of Perception. *Encyclopedia of Cognitive Science*, 2006.

[13] L. Euler. Lettres a une Princesse d'Allemagne sur divers sujets de physique et de philosophie. *Letters*, 2:102–108, 1775.

[14] A. Fish, J. Flower, and J. Howse. The Semantics of Augmented Constraint Diagrams. *Journal of Visual Languages and Computing*, 16:541–573, 2005.

[15] J. Gil, J. Howse, and S. Kent. Formalizing Spider Diagrams. In *Proceedings of IEEE Symposium on Visual Languages (VL99), Tokyo*, pages 130–137. IEEE, September 1999.

[16] C. Gurr. Effective Diagrammatic Communication: Syntactic, Semantic and Pragmatic Issues. *Journal of Visual Languages & Computing*, 10(4):317–342, August 1999.

[17] C. Gurr and K. Tourlas. Towards the Principled Design of Software Engineering Diagrams. In *Proceedings of 22nd International Conference on Software Engineering*, pages 509–518. ACM Press, 2000.

[18] E. Hammer. *Logic and Visual Information*. CSLI, Stanford, 1995.

[19] E. Hammer and S. J. Shin. Euler's Visual Logic. *History and Philosophy of Logic*, pages 1–29, 1998.

[20] D. Harel. Statecharts: A Visual Formalism for Complex Systems. *Science of Computer Programming*, 8(3):231–274, June 1987.

[21] S. Hitchman. The Details of Conceptual Modelling Notations are Important - A Comparison of Relationship Normative Language. *Communications of the Association for Information Systems*, 9(1), 2002.

[22] J. Howse, F. Molina, S. J. Shin, and J. Taylor. On Diagram Tokens and Types. In *Proceedings of 2nd International Conference on the Theory and Application of Diagrams*, pages 146–160, Georgia, USA, April 2002. Springer.

[23] J. Howse, F. Molina, and J. Taylor. SD2: A Sound and Complete Diagrammatic Reasoning System. In *Proceedings VL 2000: IEEE Symposium on Visual Languages, Seattle, USA*, pages 127–136. IEEE, 2000.

[24] J. Howse, F. Molina, J. Taylor, and S. Kent. Reasoning with Spider Diagrams. In *Proceedings of IEEE Symposium on Visual Languages (VL99), Tokyo*, pages 138–147. IEEE, September 1999.

[25] J. Howse, G. Stapleton, and J. Taylor. Spider Diagrams. *LMS Journal of Computation and Mathematics*, 8:145–194, 2005.

[26] D. N. Jansen, H. Hermanns, and J. Katoen. UML 2003 - The Unified Modeling Language. In *UML 2003 - The Unified Modeling Language*, pages 76–91. Springer, 2003.

[27] C. John, A. Fish, J. Howse, and J. Taylor. Exploring the notion of clutter in Euler diagrams. In *4th International Conference on the Theory and Application of Diagrams*, pages 267–282, Stanford, USA, 2006. Springer.

[28] S. Kent. Constraint Diagrams: Visualizing Invariants in Object Oriented Modelling. In *Proceedings of OOPSLA97*, pages 327–341. ACM Press, October 1997.

[29] C. Legg. What is a Logical Diagram? In A. Moktefi and S. Shin, editors, *Visual Reasoning with Diagrams*, Studies in Universal Logic, pages 1–18. Springer Basel, 2013.

[30] A. Moktefi. Is Euler's Circle a Symbol or an Icon? *Sign Systems Studies*, 43(4):597+, December 2015.

[31] A. Moktefi and S-J. Shin. A History of Logic Diagrams. In Dov M. Gabbay, Francis J. Pelletier, and John Woods, editors, *Handbook of the History of Logic*, volume 11, pages 611–682. Elsevier, 2012.

[32] I. Oliver, J. Howse, and G. Stapleton. Protecting Privacy: Towards a Visual Framework for Handling End-user Data. In *VL/HCC'13*, pages 67–74, 2013.

[33] C. S. Peirce. *Collected Papers*, volume 4. Harvard University Press, 1933.

[34] A. Pietarinen. Extensions of euler diagrams in peirce's four manuscripts on logical graphs. In M. Jamnik, Y. Uesaka, and S. Elzer Schwartz, editors, *Diagrammatic Representation and Inference: 9th International Conference, Diagrams 2016, Philadelphia, PA, USA, August 7-10, 2016, Proceedings*, pages 139–154, Cham, 2016. Springer.

[35] D. Roberts. *The Existential Graphs of Charles S. Peirce (Approaches to Semiotics)*. De Gruyter Mouton, 1 edition, October 1973.

[36] Y. Sato, K. Mineshima, and R. Takemura. Interpreting Logic Diagrams: A Comparison of Two Formulations of Diagrammatic Representations. In *Proceedings of the 33rd Annual Meeting of the Cognitive Science Society, CogSci 2011, Boston, USA, 2011*, 2011.

[37] A. Shimojima. Logical reasoning with diagrams. In Gerard Allwein and Jon Barwise, editors, *Logical Reasoning with Diagrams*, chapter Operational Constraints in Diagrammatic Reasoning, pages 27–48. Oxford University Press, Inc., New York, NY, USA, 1996.

[38] A. Shimojima. Inferential and Expressive Capacities of Graphical Representations: Survey and Some Generalizations. In *Proceedings of 3rd International Conference on the Theory and Application of Diagrams*, volume 2980 of *LNAI*, pages 18–21, Cambridge, UK, 2004. Springer.

[39] S. J. Shin. *The Logical Status of Diagrams*. CUP, 1994.

[40] J. F. Sowa. *Conceptual Structures: Information Processing in Mind and Machine*. Systems Programming Series. Addison-Wesley, July 1983.

[41] G. Stapleton and A. Delaney. Towards Overcoming Deficiencies in Constraint Diagrams. In *Symposium on Visual Languages and Human-Centric Computing*, pages 33–40. IEEE, 2007.

[42] G. Stapleton, J. Howse, P. Chapman, A. Delaney, J. Burton, and I. Oliver. Formalizing Concept Diagrams. In *Visual Languages and Computing*, pages 182–187. Knowledge Systems Institute, 2013.

[43] G. Stapleton, J. Howse, and J. Taylor. A Constraint Diagram Reasoning System. In *Proceedings of International Conference on Visual Languages and Computing*, pages 263–270. Knowledge Systems Insitute, 2003.

[44] G. Stapleton, J. Taylor, J. Howse, and S. Thompson. The Expressiveness of Spider Diagrams Augmented with Constants. *Submitted to Journal of Visual Languages and Computing*, 2005.

[45] F. Stjernfelt. Diagrams as a Centrepiece of a Peircean Epistemology. *Transactions of the Charles S. Peirce Society*, 36(3):357–384, 2000.

[46] F. Stjernfelt. Two iconicity notions in peirce's diagrammatology. In H Schärfe, P Hitzler, and P Øhrstrøm, editors, *Conceptual Structures: Inspiration and Application: 14th International Conference on Conceptual Structures, ICCS 2006*, pages 70–86. Springer Berlin Heidelberg, Berlin, Heidelberg, 2006.

[47] F. Stjernfelt. On Operational and Optimal Iconicity in Peirce's Diagrammatology. *Semiotica*, 2011(186), January 2011.

[48] F Stjernfelt. Dicisigns. *Synthese*, 192(4):1019–1054, 2015.

[49] J. Venn. On the Diagrammatic and Mechanical Representation of Propositions and Reasonings. *Phil.Mag*, 1880.

[50] J. Zeman. A System of Implicit Quantification. *Journal of Symbolic Logic*, 32:480–504, February 1968.

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