

# Growth processes of high-growth firms as a four-dimensional chicken and egg

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## Abstract

This article investigates whether high-growth firms grow in different ways from other firms. Specifically, we analyze how firms grow along several dimensions (growth of sales, employment, assets, and operating profits) using Structural Vector Autoregressions. Causal relations are identified by using information contained in the (non-Gaussian) growth rate distributions. For most firms, the growth process starts with employment growth, which is then followed by sales growth, then growth of operating profits, and finally growth of assets. In contrast, high growth firms put more emphasis on growth of operating profits driving other dimensions of growth, with employment growth occurring at the end.

**JEL classification:** L25; L23; D22

## 1. Introduction

Yes, there are two paths you can go by, but in the long run, there's still time to change the road you're on.  
Led Zeppelin, Stairway to Heaven, 1971

A recent finding in the empirical industrial organization literature is the peculiar distribution of firm growth rates, which is far from Gaussian and instead resembles the symmetric exponential or Laplace distribution (Stanley *et al.*, 1996; Bottazzi and Secchi, 2006). This relatively new stylized fact is remarkably robust across countries, years, sectors, and also growth rate indicators (among a large literature, see e.g. Bottazzi *et al.*, 2002; Fagiolo and Luzzi, 2006; Bottazzi *et al.*, 2010; Duschl and Peng, 2015; and Yu *et al.*, 2015). The intuition behind this “tent-shaped” distribution of growth rates is that, while most firms do not grow, a handful of firms experience either very fast decline or very fast growth. Indeed, it has been suggested that the dynamics of industries is driven not by the stagnant majority, but by a handful of outliers—“market selection seems to operate quite gently, if at all, vis-à-vis most ‘near-average’ agents. . . selection dynamics are primarily driven by outliers” (Bottazzi *et al.*, 2002: 720; see also Metcalfe, 2005).

In parallel to empirical investigations of the growth rate distribution, there has been an increasing emphasis of research on a small number of HGFs or high-growth firms (see Henrekson and Johansson, 2010, for a survey; see also Coad *et al.*, 2014 for an introduction to the recent special issue on HGFs of Industrial and Corporate Change).

HGFs are sought out by policy-makers and business scholars alike, because they make a disproportionately large contribution to job creation, productivity growth, and economic growth. However, it is notoriously difficult to pick out, *ex ante*, which firms will ultimately become HGFs. There do not seem to be any readily observable characteristics that help to distinguish between HGFs and other firms (Shane, 2009), and furthermore HGFs have a remarkable lack of persistence in their growth performance (Holzl, 2014; Daunfeldt and Halvarsson, 2015). In other words, fast growth in the previous years does not imply that the firm will continue to grow—instead sustained growth performance appears to be as likely as sustained superior performance in a game of chance (such as coin toss; Coad *et al.*, 2013). The difficulty in identifying HGFs *ex ante*, as well as the lack of persistence in high-growth episodes, is a hindrance to policy-makers who attempt to provide support for HGFs (Shane, 2009).

In this article, we take a different approach: instead of attempting to identify which firms will become HGFs, we seek to better understand the growth process in terms of how different growth dimensions coevolve, and moreover we investigate the possible existence of differences in growth processes between HGFs and other firms. It is reasonable to expect that HGFs grow in different ways from other firms because they might have higher growth ambitions, or their efforts to grow fast may make them especially vulnerable to certain growth obstacles such as insufficient demand, lack of suitable employees, or possible financial constraints. On the one hand, an increase in demand (i.e. sales growth) might be the trigger that allows HGFs to launch into a period of high growth. Alternatively, it could be that the availability of human resources might allow the firm to enact its growth plans. Another possibility might be that an increase in profits might allow the firm to signal its credibility on financial markets and receive financial resources that are critical for financing growth projects. Finally, it could be that audacious firms, with ambitions of rapid growth, must first take the risk of investing in the new assets that will be required to operate at a larger scale of operations. Indeed, there are many possible growth paths that might be associated with HGFs. Given the exploratory nature of our investigations, it would be premature to discuss all possible combinations of growth sequences in a hypotheses section (Helfat, 2007). In this article, we consider the growth processes of HGFs to be an empirical question that requires investigation.

In doing so we do not explore *which* firms grow, or *why* they grow, but *how* they grow (following McKelvie and Wiklund, 2010). Although sales growth and employment growth have often been used, individually and interchangeably (see Davidsson and Wiklund, 2001), it has become increasingly evident that these measures are not equivalent. Indeed, while the two measures of growth are correlated, the correlation is weak (Brännback *et al.*, 2014), and then not contemporaneous. Our analysis therefore examines the sequence and relationship between the growth of sales, employment, operating profits, and assets, first for our full sample of UK firms and second for the subsample of highest-growth firms. This is an explicit attempt to address the concerns of Boyd *et al.* (2005) about the over-reliance in management research of single indicators (see also Miller *et al.*, 2013). In doing so we hope to provide evidence that can contribute to the development and extension of theory relating to firm growth.

We unravel the growth processes of firms by identifying the distinct causal relationships between different growth indicators. This is done by applying Structural Vector Autoregressions (SVARs) to our data set, which analyze the coevolutionary dynamics of a set of variables that are inter-related. Our SVAR is identified not through theoretical assumptions, nor through instrumental variables, but through a data-driven approach to causal discovery, which exploits the non-Gaussian nature of the firm growth distribution to identify the latent causal ordering (Shimizu *et al.*, 2006). In particular, we build upon the Linear Non-Gaussian Acyclic Model (LiNGAM) introduced in a cross-sectional context by Shimizu *et al.* (2006), and extended to a SVAR context, by introducing lagged effects, by Moneta *et al.* (2013). This VAR-LiNGAM approach to obtaining causal estimates from observational data is often applied in the neuroimaging and machine learning literature, although it has recently been introduced into the econometrics literature by Moneta *et al.* (2013).

We therefore contribute to the literature on firm growth by applying a SVAR model that delivers causal estimates instead of mere intertemporal associations (e.g. the Vector Autoregression [VAR] models in Coad, 2010; Coad *et al.*, 2011; Colombelli *et al.*, 2014). While Moneta *et al.* (2013) applied our SVAR model to firm growth and R&D expenditure, we include data on growth of assets, and—importantly—we distinguish between subsamples of HGFs and non-HGFs to gain insights into how HGFs grow.

## 2. Data

The UK constitutes an interesting case for investigating the growth processes of HGFs, because a recent international comparison of 11 industrialized countries highlighted that the frequency of HGFs is particularly high in the UK

**Table 1.** Summary statistics for 2003 (i.e. the first year of the sample), including two-sample *t*-tests of unequal variances

	Non-HGFs			HGFs			<i>t</i> -test P-values
	Mean	SD	Observation	Mean	SD	Observation	
Employment	492.28	5224.58	17,924	43.86	181.98	1241	0.000
Sales	75,347.98	810,654.00	17,330	20,968.98	167,605.70	1160	0.000
Total assets	66,486.41	167,9991.00	21,004	18,544.61	166,541.80	1697	0.000
Operating Profits	−2105.04	237,904.90	19,312	1182.00	27,212.97	1390	0.077
Age	15.96	19.45	24,042	6.74	13.88	2476	0.000

(Bravo, 2010). For our analysis we use data on UK businesses from Bureau van Dijk's FAME (Financial Analysis Made Easy) database. FAME aggregates data from Companies House and other sources to be probably the most comprehensive private source of firm data in the UK, with the vast majority of firms in the data set being unlisted on stock markets. Our key variables, Turnover, Net Tangible Assets, and Operating Profit, are defined in terms of thousands of GBP, and for number of employees we take the headcount of employees. We take growth of operating surplus (i.e. "operating profit") as an indicator of the financial performance of the firm because it excludes non-operating expenses and taxes, etc. (Bottazzi *et al.*, 2010; Coad, 2010). However, we are aware that financial performance variables can sometimes be unreliable proxies for the underlying economic phenomena of interest (Fisher and McGowan, 1983), and therefore should be treated with some caution.

We focus on the years 2003–2011, although many of the firms in our analysis do not report data for the full period,<sup>1</sup> meaning that we have an unbalanced panel.<sup>2</sup> Table 1 presents summary statistics on our initial sample in the first observed year. If we have too few observations to observe a firm over a 3-year period, then these firms are dropped from our analysis.

In line with previous work,<sup>3</sup> we focus on firms with 20 employees or more. Including smaller firms would amplify difficulties of missing observations and hence selection bias. Instead, we focus on firms with 20 employees or more, and so our results should be interpreted accordingly. In our subsequent analysis, we sometimes split the sample into subsamples of HGFs versus non-HGFs. This is done in the following way: first we calculate a firm's average annual employment growth over the available period (with a minimum of at least 3 years). If we consider that a firm's average annual employment growth rate  $\gamma$  can be expressed in terms of the relationship between initial size  $S_t$  and final size  $S_{t+\tau}$ :

$$(1 + \gamma)^\tau = S_{t+\tau}/S_t \quad (1)$$

then the average annual growth rate  $\gamma$  can be calculated in the following way:

$$(S_{t+\tau}/S_t)^{1/\tau} - 1 = \gamma \quad (2)$$

The OECD-Eurostat definition of HGFs requires that firms have an average annual growth rate of  $\gamma \geq 20\%$  over a 3-year period (with 10+ employees in the base year, Eurostat-OECD, 2007). However, in this article, we define HGFs as those firms that are in the top 10% of the (average annual) employment growth rates distribution (in our sample of firms with 20+ employees). We choose this measure of HGFs to exploit the available data as best we can, by making use of all available years (maximum duration: 2003–2011). Firms in our sample are present for different lengths of time, and so we normalize by calculating the average annual growth rate (with a minimum of 3 years). It has been

- 1 Previous work by Cowling *et al.* (2008) shows that missing data in FAME is effectively random, and there is no evidence of any pattern in missing data.
- 2 Although we do not restrict firms to be present in each year 2003–2011, we do have the restriction that there are no gaps in the four SVAR variables for those years where a firm does report activity for that year. For example, if we have observations for a firm-year for growth of sales, employment, and assets, but not operating profits, then this firm-year will be dropped.
- 3 For example, work on data from the French National Statistical Office (INSEE), which focuses on firms above a threshold of 20 employees (see, among others, Coad, 2007a, 2010; Bottazzi *et al.*, 2010).

observed that high-growth events display little persistence (Coad, 2007a; Parker *et al.*, 2010; Holzl, 2014; Daunfeldt and Halvarsson, 2015), and therefore we do not focus on what happens after a high-growth event, but only how firms grow during their high-growth period. Although some sectors may grow faster than others, we do not normalize by sector because we argue that a high employment growth rate is equally challenging (from an organizational point of view) whatever sector the firm operates in. We prefer relative growth to absolute growth because the latter emphasizes the growth of large firms to the detriment of the growth of smaller firms (Holzl, 2014). We also focus on the top 10% of the employment growth rates distribution to ensure that we have enough firms in the HGF category, while avoiding having too large an HGF category that might also include some relatively slow-growth firms.

### 3. Methodology

#### 3.1 Background

Previous work has recognized that firm growth indicators (sales, profits, employment, etc) are not perfectly correlated with each other, but shed light on different facets of firm growth, and correspond to different economic concepts (Shepherd and Wiklund, 2009; Miller *et al.*, 2013). Achtenhagen *et al.* (2010) survey the firm growth literature and write that (p. 307):

A crucial challenge for the future study of growth lies in how to capture this complexity and multidimensionality, e.g. by not treating growth as dependent variable but as intermediary variables while studying other outcomes, such as the improvement of performance.

And also that (p. 311):

One issue considered crucial by the entrepreneurs is clearly calling for more research on the interplay of the different growth aspects. While different studies . . . have pointed out that different growth measures are not highly statistically correlated, relationships still exist between them.

and conclude that “more quantitative work is needed” (p. 310).

In this article, we seek to address these challenges to firm growth research by considering different facets of the growth process: sales growth, employment growth, growth of assets, and growth of operating profits. We therefore contribute to the literature that considers how firms grow in terms of sales and profits (Cowling, 2004). To this end, we first apply reduced-form VAR models to the analysis of firm growth (Coad, 2010; Colombelli *et al.*, 2014) to explore the intertemporal associations between the dimensions of firm growth observed in our data set (sales, employment, assets, and profits). Although intertemporal associations can describe the evolution of firms over time, they do not identify which variable is driving the other. Correlation does not imply causality—or in everyday language “you can’t get an ‘ought’ from an ‘is’.” Knowledge of the causal relations (as opposed to mere associations) is essential as soon as one wishes to consider how to intervene in the system being observed.

#### 3.2 Our SVAR estimator

To gain an understanding of causal relations, we then apply SVARs—to be precise, we apply a Linear non-Gaussian Acyclic SVAR model (VAR-LiNGAM; see Shimizu *et al.*, 2006; Hoyer *et al.*, 2006; Hyvärinen *et al.*, 2010) that is identified through Independent Component Analysis (ICA; see Hyvärinen *et al.*, 2001; Stone, 2004). We implement the algorithm in Moneta *et al.* (2013), which uses ICA to recover the latent components that are fully statistically independent, before they are arranged in the causal ordering that best fits the data. We begin by estimating a reduced-form VAR to obtain the residuals, then we apply ICA to decompose these residuals into statistically independent shocks. The rows are then permuted to obtain an estimate of a lower-triangular matrix, which has zeroes along the diagonal. Further details are in Moneta *et al.* (2013), and see also Coad and Binder (2014) for an application.

The main assumption required by our VAR-LiNGAM estimator is that the SVAR residuals  $\epsilon_{it}$  are non-Gaussian. This assumption cannot be tested directly, although we do verify that the related VAR residuals are non-Gaussian. The estimator also assumes that the causal structure is acyclic—that there is one main direction of causality between variables, and that minor feedback loops (that take place within the same time period) can be ignored. This assumption is reasonable in our context, to the extent that the major direction of causality is emphasized and any possible instantaneous feedback effects, that play a relatively minor role, are pruned down to zero. Three further assumptions can also be named here, which are shared with more conventional regression estimators. The first concerns omitted variable bias—

it is assumed that there are no strong confounding variables that have been omitted from the VAR system. Bearing in mind the difficulties in finding variables that can accurately predict firm growth (Coad, 2009), we are not overly concerned about omitted variable bias. Second, VAR-LiNGAM is a linear regression model, which assumes that the relationships between the dependent and explanatory variables are linear. In view of previous work on firm growth, this assumption seems reasonable. Third, the SVAR shocks  $\epsilon_{it}$  are assumed to be independent—that is, independent across VAR series, and independent over time. This seems to be a reasonable assumption in our present context, especially considering that the SVAR shocks are independent by construction due to our ICA procedure.

### 3.3 Control variables and preprocessing

We begin by dropping all cases where we have missing observations for our four SVAR series (i.e. growth of sales, employment, assets, or operating profits). We then preprocess our SVAR growth rate series (following Coad and Binder, 2014) to remove the possible influence of control variables  $X_{it}$ . These control variables are the following:

- Lagged logarithm of firm size, to control for the stylized fact that small firms are often observed to grow faster than larger firms (Coad, 2009).<sup>4</sup>
- Lagged logarithm of firm size, squared, to account for a possible nonlinear relationship between size and growth (if e.g. small firms grow faster up to a certain size threshold, above which size no longer varies with growth rate; cf You, 1995; Sutton, 1997).
- Three-digit industry dummies, to allow for the possible influence of sector of activity on firm-level performance (see Malerba and Orsenigo, 1997; see also Srholec and Verspagen, 2012) and the possibility that firms from different sectors may have different growth rates. Three-digit sectors are defined according to the 2007 SIC industry classification system.
- “Age” of the firm, which is measured here in terms of the number of years since the date of incorporation. This measure of age is similar to Demirel and Mazzucato (2012)’s indicator of firm age as proxied by age since the firm’s Initial Public Offering. Note that the firm may have started operations before the date of incorporation, which would be a limitation of this indicator of firm age.
- Year dummies are included to control for the influence of year-specific macroeconomic effects on firm growth, that are common to all firms in the same year.

Although there are other variables that have been mentioned in the literature as having an influence on firm growth rates, the limitations of our data sample prevent us from including further variables. Nevertheless, given that firm growth is often seen as being well approximated by a random walk (Coad, 2009; Denrell *et al.*, 2015), there is no reason to suspect that omitted variables bias should play an important role in our present context.

Our preprocessing of the growth rate series involves regressing the control variables (mentioned above) on the raw growth rate series  $G_{it}$ , for firm  $i$  in year  $t$ , to obtain the vector  $\hat{g}_{it}$ .

$$G_{it} = a + b X_{it} + \hat{g}_{it} \quad (3)$$

$\hat{g}_{it}$  is then standardized (with mean 0 and standard deviation 1) to obtain the vector  $g_{it}$ .

### 3.4 SVAR estimation

Our SVAR regression equation can be written as follows:

$$g_{it} = B g_{it} + \sum_{\tau=t-s}^{t-1} \phi_{\tau} g_{i\tau} + \epsilon_{it}, \quad (4)$$

where  $g$  is a  $4 \times 1$  vector of growth rate series (growth of sales, employment, assets, and operating profits), and  $\epsilon_{it}$  is the error term. The matrix  $B$  corresponds to a lower-diagonal matrix of instantaneous effects (which indicates the contemporaneous causal orderings of variables), while  $\phi_{i\tau}$  corresponds to the matrix of intertemporal causal effects.

4 Firm size is measured using the same variable (sales, employment, assets, or operating profits) that appears as the dependent variable in this preprocessing step. For example, if we are preprocessing employment growth, then lagged size corresponds to the lagged logarithm of employees. If we are preprocessing assets growth, then lagged size corresponds to the lagged logarithm of assets.



Our reduced-form VAR is a simplified version of the SVAR regression equation, where the right-hand-side does not include the term  $B g_{it}$ . The number of lags is represented by  $s$ . While lag selection criteria such as the Akaike Information Criterion and the Bayes Information Criterion do not provide clear-cutting recommendations in firm growth applications of our SVAR model (Moneta *et al.*, 2013), we focus on a simple one-lag model, but in our subsequent robustness analysis we also present results for two- and three-lag SVAR models.

## 4. Results

### 4.1 Correlations

Table 2 contains a correlation matrix of the four VAR series. The highest pairwise correlation is between growth of sales and growth of employment, with a correlation coefficient of 0.5493. All of the four variables—growth of sales, employment, total assets, and operating profits—are positively correlated with each other, although the correlations are far from perfect (Shepherd and Wiklund, 2009; Coad, 2010; Miller *et al.*, 2013). These correlations give a preliminary view and serve as an introduction to our VAR and SVAR results. Another interesting feature is that the correlations are all below the frequently cited threshold value of  $\pm 0.70$  (Hair *et al.*, 1998), which suggests that we do not need to be overly concerned about multicollinearity in our particular context (especially considering that we have a relatively large number of observations, which should also help in identification).

### 4.2 Reduced-form VAR results

Table 3 contains the reduced-form VAR results, which are similar in spirit to those in Coad (2010). These intertemporal associations are helpful in describing the time series evolution of the VAR series, but they do not allow any causal interpretation (and hence do not provide any information about the possible effects of a policy intervention).

We begin by looking at the results for the full sample (top panel of Table 3). First of all, along the diagonal we can see the autocorrelation coefficients. Growth of employment and assets displays positive autocorrelation over time, while the growth of sales and operating profits displays negative autocorrelation (which is particularly strongly negative for growth of operating profits).<sup>5</sup> Sales growth is followed by positive changes in employment, total assets, and operating profits, while employment growth is positively associated with subsequent growth of sales, operating profits, and assets. Growth of operating profits has a relatively strong positive association with subsequent growth of assets.

Comparing the results for the full sample (top panel of Table 3) with results for the subsample of HGFs, the results are generally quite similar, although a few differences can be mentioned. For HGFs, we observe a weaker association of employment growth with growth of the other variables (significant only in the case of subsequent sales growth). Nevertheless, for HGFs the positive association between assets growth and subsequent growth of sales, employment, and operating profits is stronger. Another interesting finding is that, for HGFs, growth of operating profit has no significant effect on subsequent growth of either sales or employment, although it is positively associated with subsequent growth of assets.

These reduced-form regression results give us a first insight into the intertemporal associations between the variables, although they are merely associations and not causal effects.

### 4.3 Structural VAR results

Before applying our SVAR model to our data, we first check that the residuals are non-Gaussian, which is one of the model requirements. Appendix 1 presents qq-plots (quantile–quantile plots) of the SVAR residuals, and shows that these residuals are indeed non-Gaussian. Non-Gaussianity is observed to be highly statistically significant when formal tests are applied ( $P$ -values of  $10^{-35}$  or smaller when Shapiro–Wilk and Shapiro–Francia tests are applied). Similar qq-plots are obtained for the HGF and non-HGF subsamples. This indicates that our SVAR identification strategy that applies ICA is an appropriate technique for our data.

5 Operating profits can fluctuate considerably over time, and (unlike the three other SVAR variables) can take negative values, which might help explain its strong negative autocorrelation. Note that a relatively strong negative autocorrelation for operating profits has been found in other VAR models (e.g. Coad, 2010; Coad *et al.*, 2011; Moneta *et al.*, 2013).

**Table 2.** Summary statistics of growth rates after preprocessing (i.e. growth rates  $g_{it}$  from equation (4)), and matrices of Pearson and Spearman correlation coefficients for the VAR series. 101,256 observations. All correlations are statistically different from zero at the 5% level

	Mean	SD	Median	Skewness	Kurtosis	Observation	Pearson correlation coefficients			Spearman's rank correlation coefficients			
							Sales growth	Employment growth	Assets growth	Profits growth	Sales growth	Employment growth	Assets growth
Sales growth	0.0000	1.0000	0.0051	-1.6707	72.1184	101,256	1						
Employment growth	0.0000	1.0000	-0.0042	-2.1646	124.0168	101,256	0.5493	1		0.4648	1		
Assets growth	0.0000	1.0000	-0.0060	-1.6302	41.8644	101,256	0.1723	0.1114	1	0.2402	0.1598	1	
Profits growth	0.0000	1.0000	0.0768	-0.7076	9.0322	101,256	0.339	0.1282	0.175	0.4386	0.1458	0.2917	1

**Table 3.** Reduced-form VAR estimates and t-statistics, estimated using Least Absolute Deviation regressions (as opposed to conventional OLS). One lag only is included in the VAR. A constant term is included in the estimations but not reported in the tables. Each row corresponds to a median regression, with the dependent variable on the left, and the regression statistics (pseudo-R2 and number of observations) on the right. Standard errors are obtained after 100 bootstrap replications

Full sample	Sales growth	Employment growth	Assets growth	Profits growth	Pseudo-R2	Observation
Sales growth	−0.0163*** (0.00460)	0.0987*** (0.00527)	0.0119*** (0.00234)	0.00245 (0.00286)	0.0063	69,950
Employment growth	0.0691*** (0.00337)	0.0639*** (0.00502)	0.0147*** (0.00181)	0.0186*** (0.00163)	0.0194	69,950
Assets growth	0.0248*** (0.00239)	0.0110*** (0.00277)	0.00898** (0.00400)	0.0307*** (0.00209)	0.0050	69,950
Profits growth	0.0539*** (0.00513)	0.0232*** (0.00513)	0.00960*** (0.00354)	−0.203*** (0.00803)	0.0177	69,950
HGFs						
Sales growth	−0.0303 (0.0185)	0.0496** (0.0222)	0.0659*** (0.0145)	−0.00810 (0.0220)	0.0067	2263
Employment growth	0.0146 (0.0194)	−0.00748 (0.0148)	0.0668*** (0.0211)	−0.00273 (0.0254)	0.0049	2263
Assets growth	−0.0105 (0.0136)	0.00319 (0.0115)	0.0249 (0.0323)	0.0686*** (0.0173)	0.0039	2263
Profits growth	0.0283* (0.0166)	0.00934 (0.0147)	0.0437*** (0.0145)	−0.176*** (0.0354)	0.0169	2263
Non-HGFs						
Sales growth	−0.0222*** (0.00715)	0.0771*** (0.00677)	0.00943*** (0.00307)	0.000977 (0.00361)	0.0030	41,408
Employment growth	0.0677*** (0.00518)	0.0362*** (0.00641)	0.0101*** (0.00258)	0.0164*** (0.00272)	0.0144	41,408
Assets growth	0.0319*** (0.00365)	0.00897* (0.00509)	0.00502 (0.00534)	0.0369*** (0.00287)	0.0055	41,408
Profits growth	0.0565*** (0.00639)	0.0127** (0.00539)	0.00711** (0.00329)	−0.192*** (0.00798)	0.0158	41,408

Following on from the reduced-form VAR results, we now focus on the structural VAR results that incorporate insights into causal relations and instantaneous effects (i.e. effects that occur within one period of observation, which in our case is 1 year). SVAR estimates of the matrix of instantaneous effects (matrix B in equation (4)) and also the matrix of the first SVAR lag ( $\phi_1$ ) are presented in Table 4.

We begin by looking at the results for the full sample, shown in the top panel of Table 4. Our SVAR estimates suggest the following causal ordering: employment growth appears to “kick-start” the growth process, having a positive effect on sales growth and assets growth, as well as a negative effect on growth of operating profit.<sup>6</sup> Taken together, these results show that employment growth is a direct cost (hence having a negative direct effect on operating profits) although there is an important indirect channel according to which employment growth boosts sales (both an instantaneous and a lagged effect), and sales growth will boost profits (again, both an instantaneous and a lagged effect). Following on from employment growth, sales growth has a positive causal effect on assets growth and operating profits. Growth of operating profits has a positive causal effect on growth of assets, while growth of assets has no direct instantaneous effect on any of the other variables.<sup>7</sup> These insights into the causal ordering of the variables

6 The table should be read as follows: the dependent variable (on the left of the table) is explained by the variables in matrices B and  $\phi_1$ . Employment growth has a positive causal effect on sales growth (coeff. = 0.52005) and on assets growth (coeff. = 0.02993) and a negative causal effect on profits growth (coeff. = −0.00688).

7 Table 4 also shows a small *negative* causal effect of profits growth on employment growth. Further robustness analysis (where HGFs are defined in terms of sales growth rather than employment growth) confirms this small negative effect.



**Table 4.** Structural VAR estimates using the VAR-LiNGAM algorithm: coefficients and standard errors

	B				$\phi_1$			
	Sales growth	Employment growth	Assets growth	Profits growth	Sales growth	Employment growth	Assets growth	Profits growth
<b>Full sample</b>								
Sales growth	0	0.5201	0	0	-0.0522	0.0655	0.0042	-0.0072
	0	0.0184	0	0	0.0045	0.0047	0.0022	0.0023
Employment growth	0	0	0	0	0.0691	0.0639	0.0147	0.0186
	0	0	0	0	0.0039	0.0059	0.0019	0.0018
Assets growth	0.1119	0.0299	0	0.1439	0.0168	-0.0053	0.0058	0.0590
	0.0081	0.0076	0	0.0053	0.0024	0.0027	0.0041	0.0023
Profits growth	0.4073	-0.0865	0	0	0.0665	-0.0115	0.0060	-0.2022
	0.0085	0.0069	0	0	0.0050	0.0047	0.0030	0.0069
<b>HGFs</b>								
Sales growth	0	0	0.2018	0.6725	-0.0472	0.0427	0.0315	0.0965
	0	0	0.0465	0.0709	0.0188	0.0204	0.0182	0.0311
Employment growth	0.7373	0	0.0301	-0.0791	0.0395	-0.0435	0.0209	-0.0128
	0.0429	0	0.0489	0.0407	0.0219	0.0208	0.0162	0.0266
Assets growth	0	0	0	0.3603	-0.0207	-0.0002	0.0092	0.1321
	0	0	0	0.0313	0.0122	0.0115	0.0287	0.0212
Profits growth	0	0	0	0	0.0283	0.0093	0.0437	-0.1762
	0	0	0	0	0.0169	0.0155	0.0145	0.0340
<b>Non-HGFs</b>								
Sales growth	0	0.4802	0	0	-0.0547	0.0597	0.0046	-0.0069
	0	0.0244	0	0	0.0072	0.0067	0.0027	0.0035
Employment growth	0	0	0	0	0.0677	0.0362	0.0101	0.0164
	0	0	0	0	0.0047	0.0062	0.0026	0.0024
Assets growth	0.1335	0.0208	0	0.1502	0.0250	-0.0040	0.0025	0.0653
	0.0113	0.0101	0	0.0068	0.0040	0.0052	0.0059	0.0030
Profits growth	0.4464	-0.0837	0	0	0.0720	-0.0186	0.0038	-0.1910
	0.0126	0.0109	0	0	0.0067	0.0057	0.0030	0.0084

could not have been obtained from reduced-form VARs (such as those in Section 2) because these latter focus only on intertemporal associations and have no way of identifying causal relations within the period.

The results for the first lag of coefficients from the SVAR model are generally similar to those emerging from the SVAR model's matrix of contemporaneous effects (i.e. the  $B$  matrix from equation (4) corresponding to "lag zero"), although there are more observed relationships between the variables (because the first-lag matrix does not have any empty cells).

It should also be noted that the results for the full sample include not only growing firms but declining firms. The processes of decline would be a mirror image of the processes of growth: first employment declines, then sales declines, followed by a fall in operating profits and then a fall in assets.

With regards to the results for the subsample of HGFs, we observe that the results are indeed different. HGFs seem to follow their own style of growth process. For HGFs, profits growth is the "initiator" of growth of the other variables. Profits growth appears to have a positive causal effect on growth of sales and assets. This is consistent with capital rationing in external markets, making the firm more dependent on internal funds to invest in growth. Then, growth of total assets has a positive effect on growth of sales (and an insignificant effect on employment growth).

This puzzling result suggests that firms that enjoy growth of profits will reduce their employment growth. Whatever the explanation, it suggests that we cannot assume that firms will necessarily plough back their profits into employment growth.

Growth of sales has a positive effect on growth of employment. Growth of employment comes last in the causal ordering.

These insights into the growth processes of HGFs are reminiscent of findings by [Achtenhagen \*et al.\* \(2010: 308\)](#), who write that: “How entrepreneurs view an increase in employment appears to be rather drastically different from what politicians would like to see.” We observe that HGFs grow by first experiencing growth of operating profits, then growth of assets, then growth of sales, with growth of employment coming last. Our results for HGFs are also reminiscent of the previous suggestion by [Davidsson \*et al.\* \(2009: 388\)](#) that “sound growth usually starts with achieving sufficient levels of profitability,” rather than vice versa. In addition, growth of operating profits (and also growth of assets and sales) may be a signal of overall firm quality, that endows managers and investors with the confidence to engage in growth, and provides the firm with the resources to pay higher wages to higher quality new employees ([Dahl and Klepper, 2015](#)). Our results show that growth of operating profits has a negative direct effect on employment growth, although this is offset by positive indirect effects via sales growth and assets growth.

#### 4.4 Robustness

Firms in our samples may display heterogeneity in their causal orderings, such that the most commonly observed causal ordering does not always represent how firms grow (in other words, even if sales growth precedes profits growth for most firms, there may be a large minority of firms for whom profits growth precedes sales growth, which would cast doubt on the robustness of the results). To investigate the prevalence of alternative causal orderings, we follow the procedure in [Duschl and Brenner \(2013\)](#), where we check the frequency of causal orderings observed for 500 bootstrapped replications (see Appendix 2 below). For the sample of non-HGFs, the observed causal ordering occurs in virtually all of the cases. For the sample of HGFs, however, we observe a number of different causal orderings. In most cases growth of operating profits comes first, and employment growth comes last. In most cases, growth of assets comes second in the causal ordering, but there are some cases where growth of sales comes second in the causal ordering.

To further investigate the robustness of our analysis, we present results for SVAR models with longer lags. Taking longer lags offers a richer econometric model at the cost of a smaller number of observations. Appendices 3 and 4 contain the SVAR results for two and three lag models, respectively. Overall, our results are similar to those from the baseline one-lag model.

We also investigated several other concerns about the robustness of our results.

First, the avid reader will recall that HGFs were defined in terms of employment growth. Do our main results hold using an alternative definition of HGFs based on sales growth (instead of employment growth)? Interestingly, we observed that our results were overall similar, although growth of sales and employment sometimes switched places in the causal ordering. For the sample of (sales) HGFs, the most frequently observed causal ordering was growth of profits, then assets, then sales, then employment (i.e. the same as for employment HGFs, although bootstrapping analysis revealed that sales and employment growth sometimes switched places). For the sample of non-HGFs, however, the most frequently observed causal ordering was growth of sales, then employment, then profits, then assets. For HGFs, defined either in terms of sales or employment, growth of profits and assets came at the start of the causal ordering, while the reverse was true for non-HGFs. Overall, these findings suggest that growth of sales and employment are closely related (as hinted at in our correlation matrix in [Table 2](#)) and that the causal ordering of these two variables (between themselves) depends on how HGFs are defined. Second, a possible concern relates to how the growth mode (i.e. organic growth vs. growth through mergers and acquisitions [M&A]) might affect our results. Using the available data on M&A activity in FAME (available to us for 2006–2011 only), we removed the known M&A events and repeated our analysis on the remaining subsample. The SVAR results were qualitatively similar to our main results, and yielded the same causal ordering as for our baseline sample. Third, to explore how firm size and sample composition effects might affect our analysis, we estimated our SVAR on a subsample of firms with 100+ employees, and obtained the same causal ordering and qualitatively similar results to our baseline full-sample case.

## 5. Discussion

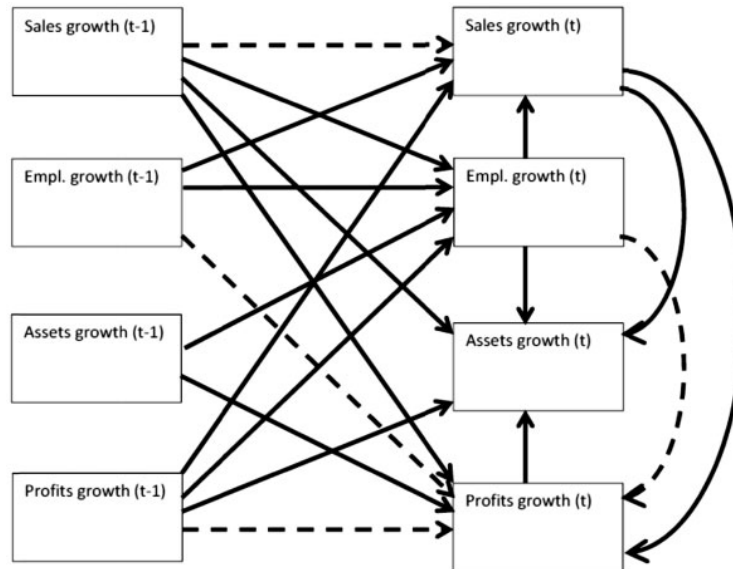
This article applies advanced econometric techniques to identify differences in the processes of growth of HGFs and the general population. In doing so, the article seeks to clarify the causal relationship between four variables commonly used to proxy firm growth: sales, employment, profits, and assets. Historically this has been difficult to do given the econometric difficulties of obtaining causal estimates from observational data on firm growth. Our work differs from previous work by not considering the questions of *which* firms grow, or *how much* they grow, but instead *how* they grow (McKelvie and Wiklund, 2010). Figures 1 and 2 summarize the SVAR results for the full sample and for the subsample of HGFs, respectively.

At the most basic level, our results for the general population of firms suggest that employment growth kick-starts the growth process, which is then followed by sales growth. This is then followed by growth of operating profits, and growth of assets occurs at the end of the causal ordering. These results are similar to those obtained previously (e.g. Coad, 2007, 2010, Moneta *et al.*, 2013) that highlight that growth of operating profits occurs toward the end of the growth process, while growth of employment and sales occur earlier on. For the full sample of all firms, the key stimulus is the addition (or subtraction) of employees, which contributes to growth of sales. Growth of operating profits is subsequently harvested, and growth of assets is determined at the end of the causal ordering. This approach follows a broadly Penrosian perspective whereby investment in staff capacity is used as the stimulus for slow expansion. Taking on new workers provides the resources to take advantage of productive opportunities, as manifested in sales growth. These sales then lead to an increase in profitability, which facilitates the firm's ability to invest these profits in assets.

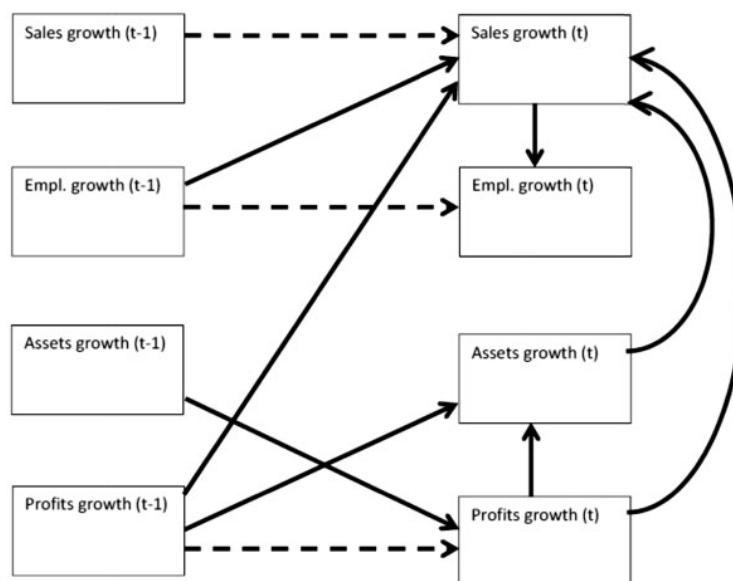
We also questioned whether or not high growth firms would act differently to the general population. Our results indicate that HGFs have different growth processes to other firms because the growth of operating profits plays a more prominent role in HGF growth. For HGFs, growth of operating profits comes first, and has a positive effect on growth of assets and sales, and a (marginally significant) negative effect on the growth of employment. Second in the causal ordering is growth of assets, suggesting that it is through growth of assets that HGFs can increase their output (and hence sales) through productivity gains. Third in the causal ordering is growth of sales, and at the end of the HGF growth process is growth of employment. Employment growth occurs relatively late in the growth process for HGFs, indicating that these firms first increase their output through physical capital-induced productivity gains before subsequently adding employees. In contrast to what was observed for firms in general, employment growth is not the initial stimulus for firm growth in the subsample of HGFs. Instead, growth of operating profits appears to be the "prime mover" for HGFs. This suggests that profits may lead to the identification of Penrose's (1959) "productive opportunity" and that the accumulation of non-staff resources using these profits is necessary to drive subsequent sales.

Our results support the argument that high growth follows a different process from growth in the general population of firms. We observe that for the general population of firms, employment growth seems to "kick-start" the growth process, with employment growth driving subsequent changes in growth of sales, assets, and profits. For HGFs, however, the causal ordering is different—first comes growth of operating profits, then assets growth, then growth of sales, and at the end comes employment growth. The growth process of HGFs puts more emphasis on growth of operating profits—HGFs may find it difficult to create jobs unless they first experience growth of operating profits, which may act as a signal to managers and investors that profitable growth opportunities are available. HGFs may be more aggressive in pursuing growth, converting profits into growth projects in circumstances where other firms would be less enterprising. For the full sample containing all firms, whether or not they show high growth, we do not observe that growth of operating profits has a causal influence on contemporaneous growth of sales or employment. In the case of non-HGFs, growth of profits comes toward the end (just before growth of assets) in the estimated causal ordering. In the case of HGFs, however, growth of operating profits comes first, and actually has a small *negative* effect on employment growth (although there are indirect positive effects via sales growth and assets growth).

Our results imply that not all firms can be pushed, or volunteer themselves, into the HGF category, by hiring large numbers of employees. Policies to stimulate HGFs should therefore not push firms to quickly hire new employees unless the underlying conditions are favorable. The opportunities and capabilities to assist high growth episodes need to be in place before implementing plans for aggressive employment growth. Simply spotting market opportunities, or taking risks regarding future market conditions, do not seem to be sufficient justification for rapid employment



**Figure 1.** Graphical representation of the SVAR results for the full sample from SVAR estimates with two time lags (reported in Table 3). Solid arrows indicate positive effects; dashed arrows indicate negative ones. Coefficients not significant at the 95% level are not shown.



**Figure 2.** Graphical representation of the SVAR results for the sample of HGFs from SVAR estimates with two time lags (reported in Table 3). Solid arrows indicate positive effects; dashed arrows indicate negative ones. Coefficients not significant at the 95% level are not shown.

growth—instead the firm needs to have already built on the opportunity, in terms of having achieved profits growth and sales growth, before following up with the hiring of new employees.

Some limitations of our analysis should also be mentioned. With regards to profits, we only observe realized profits rather than *anticipated* profits, and so we cannot comment on the possibility that it is anticipated profits that is driving the process of firm growth for the full sample of firms. Another possible data limitation is that we focused on firms with 20 or more employees, thus neglecting the growth processes of smaller firms. In future work, SVARs of the growth process might also investigate the roles of other variables in the growth process, and include other variables such as productivity or R&D expenditure. Furthermore, we focused on the 10% fastest-growing firms, but

other definitions of HGFs might yield different results. Future work might also investigate how the growth process varies over the business cycle, or for firms in different countries or institutional contexts. Our SVAR model required the assumption that the relationships between the growth variables were acyclic (i.e. one causal direction between variables with no feedback loops), but future work might allow for cyclic relations between variables (cf Lacerda *et al.*, 2008).

Our empirical analysis has made a number of important contributions, however. First, it identifies an underlying schema of the growth process for the general population of business firms, focusing on the relationships between four main performance variables. Second, it identifies a unique pattern of growth among HGFs, whereby it is growth of operating profits that is the initial stimulus that triggers subsequent growth of assets, then sales, and then employment. Third, it does all this using an advanced technique for identifying causality which has considerable potential for use in the field of Management and Industrial Organization.

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## References

- Achtenhagen, L., L. Naldi and L. Melin. (2010), 'Business growth – do practitioners and scholars really talk about the same thing?', *Entrepreneurship Theory and Practice*, 34(2), 289–316.
- Bottazzi, G., E. Cefis and G. Dosi. (2002), 'Corporate growth and industrial structures: some evidence from the Italian manufacturing industry,' *Industrial and Corporate Change*, 11(4), 705–723.
- Bottazzi, G. and A. Secchi. (2006), 'Explaining the distribution of firm growth rates,' *Rand Journal of Economics*, 37(2), 235–256.
- Bottazzi, G., A. Secchi and F. Tamagni. (2008), 'Productivity, profitability and financial performance,' *Industrial and Corporate Change*, 17(4), 711–751.
- Bottazzi, G., G. Dosi, N. Jacoby, A. Secchi and F. Tamagni. (2010), 'Corporate performances and market selection: some comparative evidence,' *Industrial and Corporate Change*, 19(6), 1953–1996.
- Bottazzi, G., A. Coad, N. Jacoby and A. Secchi. (2011), 'Corporate growth and industrial dynamics: evidence from French manufacturing,' *Applied Economics*, 43(1), 103–116.
- Boyd, B. K., S. Gove and M. A. Hitt. (2005), 'Consequences of measurement problems in strategic management research: the case of Amihud and Lev,' *Strategic Management Journal*, 26(4), 367–375.
- Brännback, M., A. L. Carsrud and N. Kiviluoto. (2014), *Understanding the Myth of High Growth Firms: The Theory of the Greater Fool*. Springer Science & Business Media. Springer: New York, Heidelberg, Dordrecht & London.
- Bravo, B. A. (2010), Growth dynamics: exploring business growth and contraction in Europe and the US, NESTA research report, November 2010. NESTA: London.
- Capasso, M., E. Cefis and K. Frenken. (2014), 'On the existence of persistently outperforming firms,' *Industrial and Corporate Change*, 23(4), 997–1036.
- Caves, R. (1998), 'Industrial organization and new findings on the turnover and mobility of firms,' *Journal of Economic Literature*, 36(4), 1947–1982.
- Coad, A. (2007a), 'A closer look at serial growth rate correlation,' *Review of Industrial Organization*, 31(1), 69–82.
- Coad, A. (2007b), 'Testing the principle of 'growth of the fitter': the relationship between profits and firm growth,' *Structural Change and Economic Dynamics*, 18(3), 370–386.
- Coad, A. (2009), *The Growth of Firms: A Survey of Theories and Empirical Evidence*. Edward Elgar: Cheltenham, UK.
- Coad, A. (2010), 'Exploring the processes of firm growth: evidence from a vector autoregression,' *Industrial and Corporate Change*, 19(6), 1677–1703.
- Coad, A. and M. Binder. (2014), 'Causal linkages between work and life satisfaction and their determinants in a structural VAR approach,' *Economics Letters*, 124, 263–268.
- Coad, A., S. O. Daunfeldt, W. Holzl, D. Johansson and P. Nightingale. (2014), 'High-growth firms: Introduction to the special issue,' *Industrial and Corporate Change*, 23(1), 91–112.
- Coad, A., J. Frankish, R. Roberts and D. Storey. (2013), 'Growth paths and survival chances,' *Journal of Business Venturing*, 28(5), 615–632.



- Coad, A. and W. Holzl. (2009), 'On the autocorrelation of growth rates,' *Journal of Industry, Competition and Trade*, **9**, 139–166.
- Coad, A. and R. Rao. (2010), 'Firm growth and R&D expenditure,' *Economics of Innovation and New Technology*, **19**(2), 127–145.
- Coad, A., R. Rao and F. Tamagni. (2011), 'Growth processes of Italian manufacturing firms,' *Structural Change and Economic Dynamics*, **22**(1), 54–70.
- Colombelli, A., J. Krafft and F. Quatraro. (2014), 'High-growth firms and technological knowledge: do gazelles follow exploration or exploitation strategies?,' *Industrial and Corporate Change*, **23**(1), 261–291.
- Cowling, M. (2003), The employment contribution of the self-employed in the EU, Research Report URN 03/539. Small Business Service. London. [www.sbs.gov.uk/research/](http://www.sbs.gov.uk/research/). <http://berr.gov.uk/files/file38300.pdf>.
- Cowling, M. (2004), 'The growth-profit nexus,' *Small Business Economics*, **22**(1), 1–9.
- Cowling, M., P. Bates, N. Jagger and G. Murray. (2008), *Study of the impact of the Enterprise Investment Scheme (EIS) and Venture Capital Trusts (VCTs) on company performance*, HMRC Research Report 44. HMRC: London.
- Dahl, M. S. and S. Klepper. (2015), 'Whom do new firms hire?,' *Industrial and Corporate Change*, **24**(4), 819–836.
- Daunfeldt, S. O. and D. Halvarsson. (2015), 'Are high-growth firms one-hit wonders? Evidence from Sweden,' *Small Business Economics*, **44**, 361–383.
- Davidsson, P. and J. Wiklund. (2001), 'Levels of analysis in entrepreneurship research: Current research practice and suggestions for the future,' *Entrepreneurship Theory and Practice*, **25**(4), 81–100.
- Davidsson, P., P. Steffens and J. Fitzsimmons. (2009), 'Growing profitable or growing from profits: Putting the horse in front of the cart?,' *Journal of Business Venturing*, **24**, 388–406.
- Delmar, F., A. McKelvie and K. Wennberg. (2013), 'Untangling the relationships among growth, profitability and survival in new firms,' *Technovation*, **33**(8–9), 276–291.
- Demirel, P. and M. Mazzucato. (2012), 'Innovation and firm growth: is R&D worth it?,' *Industry and Innovation*, **19**(1), 45–62.
- Denrell, J., C. Fang and C. Liu. (2015), Chance explanations in the management sciences,' *Organization Science*, in press.
- Duschl, M. and T. Brenner. (2013), 'Growth dynamics in regional systems of technological activities,' *Working Papers on Innovation and Space*, University of Marburg.
- Duschl, M. and S. Peng. (2015), 'The patterns of Chinese firm growth: a conditional estimation approach of the asymmetric exponential power density,' *Industrial and Corporate Change*, in press.
- Eurostat-OECD (2007), *Eurostat-OECD Manual on Business Demography Statistics*. Office for Official Publications of the European Communities: Luxembourg.
- Evans, P. and R. Welpton. (2009), 'Methods explained – business structure database,' *Economic and Labour Market Review*, **3**(6), 71–75.
- Fagiolo, G. and A. Luzzi. (2006), 'Do liquidity constraints matter in explaining firm size and growth? Some evidence from the Italian manufacturing industry,' *Industrial and Corporate Change*, **15**(1), 1–39.
- Fisher, F. and J. McGowan. (1983), 'On the misuse of accounting rates of return to infer monopoly profits,' *American Economic Review*, **73**(1), 82–97.
- Garnsey, E., E. Stam and P. Heffernan. (2006), 'New firm growth: Exploring processes and paths,' *Industry & Innovation*, **13**(1), 1–20.
- Geroski, P. A. (2000), 'The growth of firms in theory and in practice,' in N. Foss and V. Mahnke (eds), *Competence, Governance and Entrepreneurship*. Oxford University Press, pp. 168–186.
- Goddard, J., D. McMillan and J. O. S. Wilson. (2006), 'Do firm sizes and profit rates converge? Evidence on Gibrat's Law and the persistence of profits in the long run,' *Applied Economics*, **38**, 267–278.
- Hair, J. F., R. E. Anderson, R. L. Tatham and W. C. Black. (1998), *Multivariate Data Analysis*, 5th edn. Prentice Hall: Upper Saddle River, New Jersey.
- Helfat, C. E. (2007), 'Stylized facts, empirical research and theory development in management,' *Strategic Organization*, **5**(2), 185–192.
- Henrekson, M. and D. Johansson. (2010), 'Gazelles as job creators: a survey and interpretation of the evidence,' *Small Business Economics*, **35**(2), 227–244.
- Holzl, W. (2014), 'Persistence, survival, and growth: a closer look at 20 years of fast-growing firms in Austria,' *Industrial and Corporate Change*, **23**(1), 199–231.
- Hoyer, P., S. Shimizu and A. Kerminen. (2006), 'Estimation of linear, non-gaussian causal models in the presence of confounding latent variables,' Proceedings of the Third European Workshop on Probabilistic Graph Models (PGM 06), pp. 155–162. Available as Arxiv preprint cs/0603038.
- Hyvärinen, A., J. Karhunen and E. Oja. (2001), *Independent Component Analysis*, vol. 26. Wiley-interscience.
- Hyvärinen, A., K. Zhang, S. Shimizu and P. Hoyer. (2010), 'Estimation of a structural vector autoregression model using non-gaussianity,' *Journal of Machine Learning Research*, **11**, 1709–1731.
- Lacerda, G., P. Spirtes, J. Ramsey and P. Hoyer. (2008), 'Discovering cyclic causal models by independent components analysis,' Proceedings of the 24th Conference on Uncertainty in Artificial Intelligence (UAI-2008).



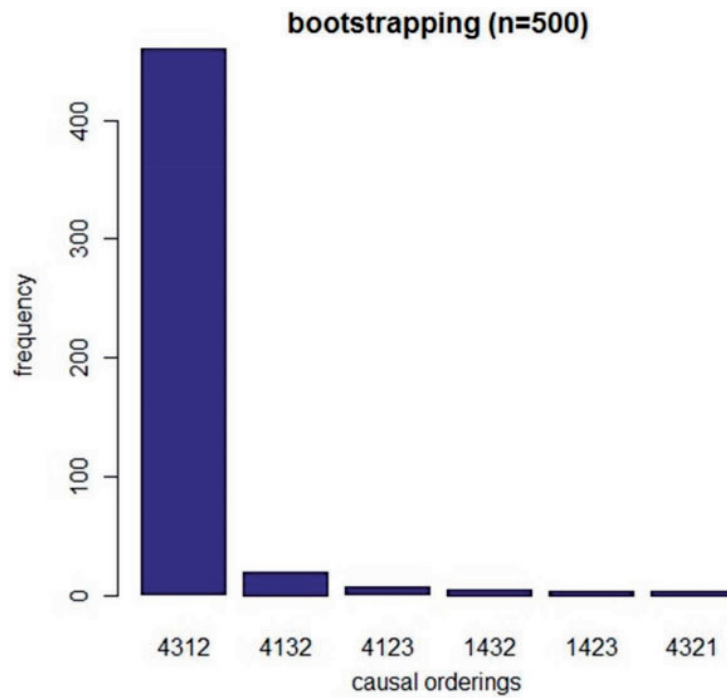
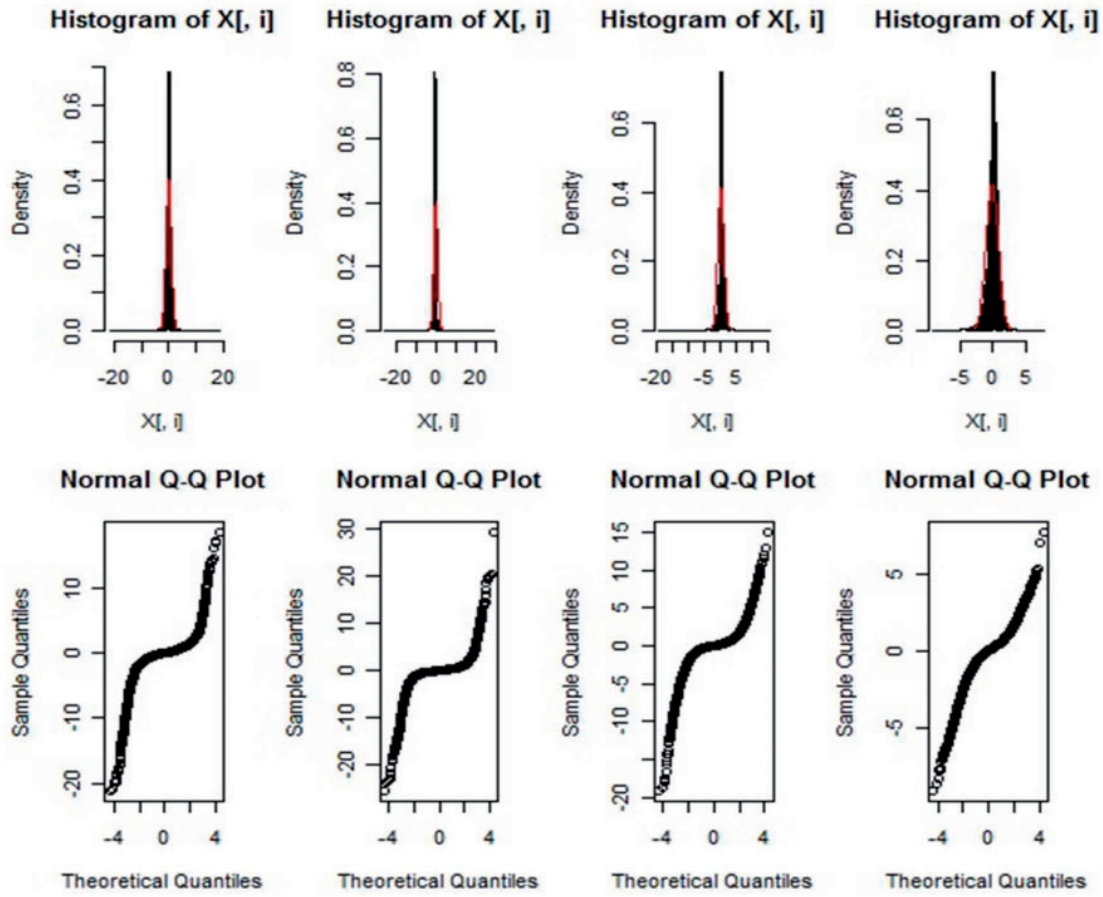
- Lee, S. (2014), 'The relationship between growth and profit: evidence from firm-level panel data,' *Structural Change and Economic Dynamics*, 28, 1–11.
- Malerba, F. and L. Orsenigo. (1997), 'Technological regimes and sectoral patterns of innovative activities,' *Industrial and Corporate Change*, 6(1), 83–117.
- Mason, C. and R. Brown. (2010), High growth firms in Scotland, Final report for Scottish Enterprise, Glasgow.
- McKelvie, A. and J. Wiklund. (2010), 'Advancing firm growth research: a focus on growth mode instead of growth rate,' *Entrepreneurship Theory and Practice*, 34(2), 261–288.
- Metcalfe, J. S. (2005), 'Accounting for evolution: an assessment of the population method,' *Working paper, 27th July 2005*, CRIC, University of Manchester
- Miller, C. C., N. T. Washburn and W. H. Glick. (2013), 'The myth of firm performance,' *Organization Science*, 24(3), 948–964.
- Moneta, A., N. Chlaß, D. Entner and P. Hoyer. (2011), 'Causal search in structural vector autoregressive models,' *Journal of Machine Learning Research: Workshop and Conference Proceedings*, 12, 95–114.
- Moneta, A., D. Entner, P. Hoyer and A. Coad. (2013), 'Causal inference by independent component analysis: theory and applications,' *Oxford Bulletin of Economics and Statistics*, 75(5), 705–730.
- Nightingale P. and A. Coad. (2014), 'Muppets and gazelles: political and methodological biases in entrepreneurship research,' *Industrial and Corporate Change*, 21(1), 113–143.
- Parker, S., D. Storey and A. van Witteloostuijn. (2010), 'What happens to gazelles? The importance of dynamic management strategy,' *Small Business Economics*, 35, 203–226.
- Penrose, E. T. (1959), *The Theory of the Growth of the Firm*. Basil Blackwell: Oxford.
- Shane, S. (2009), 'Why encouraging more people to become entrepreneurs is bad public policy,' *Small Business Economics*, 33, 141–149.
- Shepherd, D. and J. Wiklund. (2009), 'Are we comparing Apples with Apples or Apples with Oranges? Appropriateness of knowledge accumulation across growth studies,' *Entrepreneurship Theory and Practice*, 33(1), 105–123.
- Shimizu, S., P. Hoyer, A. Hyvärinen and A. Kerminen. (2006), 'A linear non-gaussian acyclic model for causal discovery,' *Journal of Machine Learning Research*, 7, 2003–2030.
- Srholec, M. and B. Verspagen. (2012), 'The Voyage of the Beagle into innovation: explorations on heterogeneity, selection, and sectors,' *Industrial and Corporate Change*, 21(5), 1221–1253.
- Stanley, M. H. R., L. A. N. Amaral, S. V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger and H. E. Stanley. (1996), 'Scaling behavior in the growth of companies,' *Nature*, 379, 804–806.
- Steffens, P., P. Davidson and J. Fitzsimmons. (2009), 'Performance configurations over time: Implications for growth- and profit-oriented strategies,' *Entrepreneurship Theory and Practice*, 33(1), 125–148.
- Stone, J. (2004), *Independent Component Analysis: A Tutorial Introduction*. MIT Press: Cambridge, MA.
- Storey, D. (1994), *Understanding the Small Business Sector*. Thomson: London, UK.
- Sutton, J. (1997), 'Gibrat's legacy,' *Journal of Economic Literature*, XXXV, 40–59.
- You, J. I. (1995), 'Small firms in economic theory,' *Cambridge Journal of Economics*, 19, 441–462.
- Yu, X., G. Dosi, J. Lei and A. Nuvolari. (2015), 'Institutional change and productivity growth in China's manufacturing: the micro-economics of knowledge accumulation and creative restructuring,' *Industrial and Corporate Change*, in press.

## Appendix 1

Quantile–quantile plots of the VAR residuals for the full sample, for the one-lag model. These plots provide justification for the VAR-LiNGAM assumption of non-Gaussian residuals. The qq-plots for subsamples of HGFs and non-HGFs, as well as for the two-lag and three-lag models, are similar.

## Appendix 2

Robustness analysis: Frequencies of heterogeneous observed causal pathways from 500 bootstrap replications. For details on the technique, see [Duschl and Brenner \(2013\)](#), see in particular their Appendix). For the full sample, and the sample of non-HGFs, for SVARs having either one lag or two lags, the results were all the same, and are not shown here: in all cases, the observed causal ordering was employment growth, then sales growth, then profits growth, then assets growth. For the full sample and the sample of non-HGFs, in the case of the three-lag SVAR, the same causal ordering was observed in the vast majority (about 98%) of the cases, but not all. For the subsample of HGFs, more heterogeneity of causal orderings across bootstrap samples was observed. The bar-chart below shows the bootstrap analysis results for a one-lag SVAR. Two-lag and three-lag SVARs showed similar patterns, with “4312” being the predominant causal ordering, although other causal orderings (e.g. “4132,” “4123,” “1432,” and “1423”) were also observed in a minority of cases.



**Figure A2.1.** Bootstrap analysis of causal orderings for HGF subsamples (one-lag SVAR). 1 = sales growth, 2 = employment growth, 3 = assets growth, 4 = profits growth.

## Appendix 3: SVAR model with two lags

	B				$\varphi_1$				$\varphi_2$			
	Sales growth	Employment growth	Assets growth	Profits growth	Sales growth	Employment growth	Assets growth	Profits growth	Sales growth	Employment growth	Assets growth	Profits growth
Full sample												
Sales growth	0	0.5089	0	0	-0.0559	0.0695	0.0064	-0.0121	-0.0122	0.0216	0.0030	-0.0189
	0	0.0224	0	0	0.0065	0.0062	0.0028	0.0034	0.0049	0.0046	0.0026	0.0032
Employment growth	0	0	0	0	0.0697	0.0451	0.0144	0.0232	0.0183	0.0101	0.0108	0.0138
	0	0	0	0	0.0049	0.0068	0.0020	0.0024	0.0032	0.0039	0.0021	0.0022
Assets growth	0.1058	0.0327	0	0.1493	0.0110	-0.0107	-0.0075	0.0744	0.0107	0.0073	0.0107	0.0460
	0.0098	0.0105	0	0.0058	0.0029	0.0029	0.0052	0.0029	0.0028	0.0031	0.0038	0.0025
Profits growth	0.4070	-0.0804	0	0	0.0804	-0.0206	0.0172	-0.2506	0.0230	0.0079	0.0070	-0.0825
	0.0105	0.0079	0	0	0.0072	0.0052	0.0046	0.0084	0.0055	0.0052	0.0036	0.0062
HGFs												
Sales growth	0	0	0.2087	0.5617	-0.0710	0.0681	0.0212	0.0868	0.0134	-0.0085	0.0252	-0.0545
	0	0	0.0668	0.0911	0.0257	0.0244	0.0244	0.0487	0.0230	0.0190	0.0249	0.0357
Employment growth	0.7528	0	0.0575	-0.1490	0.0540	-0.0614	0.0085	0.0270	-0.0116	-0.0132	0.0076	0.0878
	0.0602	0	0.0530	0.0553	0.0336	0.0346	0.0187	0.0392	0.0198	0.0153	0.0223	0.0296
Assets growth	0	0	0	0.4294	-0.0207	-0.0035	-0.0181	0.1696	-0.0408	0.0350	-0.0046	0.0890
	0	0	0	0.0447	0.0134	0.0118	0.0352	0.0309	0.0229	0.0211	0.0269	0.0307
Profits growth	0	0	0	0	0.0316	0.0067	0.0577	-0.2081	0.0032	-0.0042	-0.0155	0.0075
	0	0	0	0	0.0227	0.0170	0.0228	0.0580	0.0245	0.0179	0.0240	0.0414
Non-HGFs												
Sales growth	0	0.4824	0	0	-0.0590	0.0560	0.0086	-0.0120	-0.0199	0.0130	0.0106	-0.0161
	0	0.0313	0	0	0.0089	0.0085	0.0036	0.0044	0.0068	0.0072	0.0039	0.0036
Employment growth	0	0	0	0	0.0729	0.0283	0.0109	0.0198	0.0114	-0.0147	0.0087	0.0105
	0	0	0	0	0.0053	0.0082	0.0035	0.0025	0.0038	0.0047	0.0032	0.0025
Assets growth	0.1363	0.0076	0	0.1574	0.0218	-0.0124	-0.0133	0.0830	0.0206	0.0123	0.0104	0.0494
	0.0155	0.0124	0	0.0073	0.0054	0.0052	0.0063	0.0039	0.0042	0.0051	0.0052	0.0035
Profits growth	0.4453	-0.0697	0	0	0.0872	-0.0251	0.0122	-0.2339	0.0247	0.0034	-0.0015	-0.0698
	0.0170	0.0140	0	0	0.0099	0.0081	0.0052	0.0102	0.0068	0.0072	0.0037	0.0070

### Appendix 4. SVAR model with three lags (but only the first two lags are reported here)

	B				$\phi_1$				$\phi_2$			
	Sales growth	Employment growth	Assets growth	Profits growth	Sales growth	Employment growth	Assets growth	Profits growth	Sales growth	Employment growth	Assets growth	Profits growth
Full sample												
Sales growth	0	0.4991	0	0	-0.0685	0.0657	0.0098	-0.0140	-0.0197	0.0250	0.0091	-0.0262
	0	0.0291	0	0	0.0095	0.0101	0.0036	0.0044	0.0074	0.0070	0.0044	0.0045
Employment growth	0	0	0	0	0.0727	0.0310	0.0119	0.0229	0.0176	0.0081	0.0084	0.0105
	0	0	0	0	0.0060	0.0095	0.0030	0.0026	0.0049	0.0043	0.0027	0.0029
Assets growth	0.1097	0.0328	0	0.1426	0.0155	-0.0106	-0.0212	0.0765	0.0127	-0.0028	-0.0008	0.0574
	0.0124	0.0135	0	0.0072	0.0035	0.0040	0.0068	0.0036	0.0040	0.0039	0.0047	0.0031
Profits growth	0.4056	-0.0720	0	0	0.0783	-0.0196	0.0139	-0.2591	0.0327	0.0057	0.0070	-0.0980
	0.0143	0.0105	0	0	0.0083	0.0062	0.0057	0.0120	0.0078	0.0068	0.0047	0.0089
HGFs												
Sales growth	0	0	0.2687	0.5153	-0.0438	0.0400	0.0283	0.1042	-0.0003	-0.0042	0.0099	0.0282
	0	0	0.1081	0.1316	0.0307	0.0293	0.0346	0.0547	0.0359	0.0330	0.0428	0.0607
Employment growth	0.6782	0	0.1007	-0.1288	0.0729	-0.0692	0.0023	0.0150	0.0095	-0.0355	0.0220	0.0793
	0.0864	0	0.0816	0.0814	0.0401	0.0431	0.0263	0.0414	0.0308	0.0285	0.0292	0.0453
Assets growth	0	0	0	0.4671	-0.0148	-0.0007	-0.0409	0.1712	-0.0146	-0.0058	0.0217	0.1225
	0	0	0	0.0631	0.0165	0.0131	0.0515	0.0463	0.0244	0.0250	0.0349	0.0373
Profits growth	0	0	0	0	0.0084	0.0163	0.0472	-0.2004	-0.0187	0.0362	-0.0322	-0.0587
	0	0	0	0	0.0272	0.0212	0.0441	0.0729	0.0337	0.0243	0.0322	0.0584
Non-HGFs												
Sales growth	0	0.4676	0	0	-0.0693	0.0556	0.0088	-0.0162	-0.0294	0.0176	0.0133	-0.0273
	0	0.0391	0	0	0.0126	0.0109	0.0045	0.0058	0.0107	0.0113	0.0041	0.0058
Employment growth	0	0	0	0	0.0673	0.0154	0.0103	0.0216	0.0136	-0.0141	0.0082	0.0084
	0	0	0	0	0.0059	0.0105	0.0039	0.0031	0.0047	0.0072	0.0038	0.0036
Assets growth	0.1273	0.0075	0	0.1541	0.0250	-0.0089	-0.0210	0.0904	0.0185	0.0034	0.0010	0.0645
	0.0178	0.0150	0	0.0091	0.0058	0.0062	0.0088	0.0049	0.0061	0.0063	0.0070	0.0043
Profits growth	0.4456	-0.0401	0	0	0.0844	-0.0216	0.0085	-0.2383	0.0230	0.0037	-0.0031	-0.0770
	0.0228	0.0154	0	0	0.0127	0.0098	0.0066	0.0134	0.0102	0.0098	0.0053	0.0101