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SENSITIVITY OF THE STOCHASTIC RESPONSE OF STRUCTURES PROTECTED BY THE VIBRATING BARRIER CONTROL DEVICE

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The sensitivity of the stochastic response of a novel passive control device named Vibrating Barrier (ViBa) developed for reducing the seismic response of structures to earthquake excitation is scrutinized. The Vibrating Barrier (ViBa) is a massive structure, hosted in the soil, calibrated for protecting structures by exploiting the structure-soil-structure interaction effect. The soil is modelled as a linear elastic medium with hysteretic damping by resorting to the Boundary Element Method in the frequency domain. In order to accomplish efficient sensitivity analyses, a reduced model is determined by means of the Craig-Bampton procedure. Moreover, a lumped parameter model is used for converting the hysteretic damping soil model rigorously valid in the frequency domain to the approximately equivalent viscous damping model in order to perform conventional time-history analysis. The sensitivity is evaluated by determining a semi-analytical method based on the dynamic modification approach for the case of multi-variate stochastic input process. A non-stationary zero mean Gaussian random process is considered as stochastic input. The paper presents the sensitivity of the maximum response statistics to the design parameters of the ViBa in protecting a model of an Industrial Building. Comparisons with pertinent Monte Carlo Simulation will show the effectiveness of the proposed approach.

Keywords: Structure-Soil-Structure Interaction, hysteretic damping, Craig-Bampton method, sensitivity analysis, stochastic response, Vibrating Barriers.

1 Introduction

Unpredicted vibrations due to ground motion earthquakes cause severe damages to the structural components that lead to the deterioration or collapse of buildings. Although several techniques and strategies of Vibration Control can be adopted for the seismic design of new structures or seismic retrofit of existing buildings, every approach is based on the direct design or intervention on the members or on the control systems belonging to the structure. Conversely, for heritage buildings, strong interventions are avoidable to preserve the authenticity and integrity of the historic character of the monument; moreover, most of the existing private buildings are seismically deficient requiring an important cost impact for their seismic protection.

In this context, a novel passive control device called Vibrating Barrier (ViBa), has been recently proposed by Cacciola (2012). The Vibrating Barrier is a massive structure, hosted in the soil and detached from the other structures, calibrated for absorbing portion of the ground motion input energy. The aim is to reduce the vibrations of neighborhood structures by exploiting the structure-soil-structure interaction (SSSI) effect, i.e. the dynamic influence among vibrating structures caused by the wave propagation through the soil. To achieve this goal, the proper calibration of the ViBa parameters is required. In this framework, uncertainty plays a relevant importance in the ViBa engineering design. The uncertainties such as the random nature of the seismic action and the dispersion of the mechanical properties result in a substantial difference between the actual and the computed seismic response of the structures. Therefore, sensitivity analysis evaluating the partial derivatives of a performance measure with respect to system parameters is performed to predict the effect of the uncertainty on the structural response.

In this paper, the sensitivity of the response of a structure protected by ViBa under stochastic seismic process has been investigated. The semi-analytical modal procedure proposed by Cacciola et al. (2005), has been extended in order to consider multi-variate Gaussian stochastic load process. This method allows the evaluation of the sensitivity of the nodal response of large MDOF systems in the modal space corresponding to the nominal values. The analysis is performed on the reduced model derived by means of the Craig-Bampton procedure (Bampton and Craig 1968). The effects of interaction between the structure and the soil, namely the soil-structure interaction (SSI), are considered according to the substructure approach proposed by Kausel (1978) in which the soil is simulated by dynamic impedances subjected to seismic forces. The soil impedances are contained in the dynamic stiffness matrix computed by boundary element method (BEM) and accounts for hysteretic soil damping (Neumark 1957). In this paper, a Lumped Parameter Model (LPM) composed of frequency-independent parameters, is used for converting the exact soil-foundation reference hysteretic model formulated in the frequency domain to an approximately equivalent viscous model in the time domain. Finally, numerical sensitivity analyses are carried out for investigating the stochastic response of a model of Reactor Building with respect to the small variation of the design parameter of the ViBa.

2 Problem formulation of the global model

Consider the large global n-degree of freedom (n-DOF) structural linear system depicted in Figure 1. The dynamic governing equations of motion are casted in the frequency domain as follows:

$$\left\{\mathbf{K}_{\text{glob}}(\omega) - \omega^2\mathbf{M}_{\text{glob}} + i\omega\mathbf{C}_{\text{glob}}\right\}\mathbf{u}(\omega) = \mathbf{f}(\omega) \quad (1)$$

where $i = \sqrt{-1}$; $\mathbf{M}_{\text{glob}}$, $\mathbf{C}_{\text{glob}}$, and $\mathbf{K}_{\text{glob}}(\omega)$ are the real [n x n] global mass, damping and stiffness matrices respectively; $\mathbf{u}(\omega)$ and $\mathbf{f}(\omega)$ corresponds to the [n x 1] vectors of the nodal absolute displacements and the applied forces in the frequency domain (ω is the circular frequency).

The global system is partitioned in three subdomains or sub-structures, namely the structure to be protected hereafter referred in the paper by the subscript $[\cdot]_{\text{str}}$, the proposed device ViBa, indicated by the subscript $[\cdot]_{\text{ViBa}}$, and the soil-foundations interface denoted by $[\cdot]_{\text{SF}}$.

Therefore, Eq. (1) is restated as:

$$\left\{\begin{bmatrix} \mathbf{K}_{\mathrm{ViBa}} & \mathbf{0} & \mathbf{K}_{\mathrm{ViBa,SF}} \\ \mathbf{0} & \mathbf{K}_{\mathrm{str}} & \mathbf{K}_{\mathrm{str,SF}} \\ \mathbf{K}_{\mathrm{SF,ViBa}} & \mathbf{K}_{\mathrm{SF,str}} & \mathbf{K}_{\mathrm{SF}} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_{\mathrm{ViBa}} & \mathbf{0} & \mathbf{M}_{\mathrm{ViBa,SF}} \\ \mathbf{0} & \mathbf{M}_{\mathrm{str}} & \mathbf{M}_{\mathrm{str,SF}} \\ \mathbf{M}_{\mathrm{SF,ViBa}} & \mathbf{M}_{\mathrm{SF,str}} & \mathbf{M}_{\mathrm{SF}} \end{bmatrix} + i\omega \begin{bmatrix} \mathbf{C}_{\mathrm{ViBa}} & \mathbf{0} & \mathbf{C}_{\mathrm{ViBa,SF}} \\ \mathbf{0} & \mathbf{C}_{\mathrm{str}} & \mathbf{C}_{\mathrm{str,SF}} \\ \mathbf{C}_{\mathrm{SF,ViBa}} & \mathbf{C}_{\mathrm{SF,str}} & \mathbf{C}_{\mathrm{SF}} \end{bmatrix}\right\} \begin{bmatrix} \mathbf{u}_{\mathrm{ViBa}}(\omega) \\ \mathbf{u}_{\mathrm{str}}(\omega) \\ \mathbf{u}_{\mathrm{SF}}(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{f}_{\mathrm{SF}}(\omega) \end{bmatrix} \tag{2}$$

The vector $\mathbf{u}(\omega)$ is hence divided into the [p x 1]-vector of the ViBa, $\mathbf{u}_{\mathrm{ViBa}}$, the [q x 1]-vector of the structure, $\mathbf{u}_{\mathrm{str}}$, and the [r x 1]-vector of the soil-foundations system $\mathbf{u}_{\mathrm{SF}}$. The mass, damping and stiffness [q x q]-matrices of the structure to be protected, that is $\mathbf{M}_{\mathrm{str}}$, $\mathbf{C}_{\mathrm{str}}$, $\mathbf{K}_{\mathrm{str}}$, respectively, are derived by a traditional finite element approach. Same approach is used for the [p x p]-matrices, $\mathbf{M}_{\mathrm{ViBa}}$, $\mathbf{C}_{\mathrm{ViBa}}$, $\mathbf{K}_{\mathrm{ViBa}}$ of the proposed device, the ViBa and for the matrices related to the coupling between structure and foundations indicated by the subscript $[\cdot]_{\mathrm{SF,ViBa}}$ and by the subscript $[\cdot]_{\mathrm{SF,str}}$ and their transpose matrices indicated by the subscript $[\cdot]_{\mathrm{ViBa,SF}}$ and by the subscript $[\cdot]_{\mathrm{str,SF}}$. The [r x r]-matrices $\mathbf{M}_{\mathrm{SF}}$, $\mathbf{C}_{\mathrm{SF}}$, and $\mathbf{K}_{\mathrm{SF}}$ are the matrices of the nodes at the soil-foundations interface determined by the substructure approach proposed by Kausel (1978); by defining $\mathbf{K}_{\mathrm{dyn}}(\omega)$ as the dynamic stiffness matrix, that can be decomposed in the real part (Re) and imaginary part (Im) as:

$$\mathbf{K}_{\mathrm{dyn}}(\omega) = \mathrm{Re}\{\mathbf{K}_{\mathrm{dyn}}(\omega)\} + i\mathrm{Im}\{\mathbf{K}_{\mathrm{dyn}}(\omega)\} \tag{3}$$

the following relations are derived: $\mathbf{M}_{\mathrm{SF}} = \mathbf{M}_{\mathrm{F}}$, $\mathbf{C}_{\mathrm{SF}} = \mathbf{C}_{\mathrm{F}} + \mathrm{Im}\{\mathbf{K}_{\mathrm{dyn}}(\omega)\}/\omega$ and $\mathbf{K}_{\mathrm{F}} + \mathrm{Re}\{\mathbf{K}_{\mathrm{dyn}}(\omega)\}$; $\mathbf{M}_{\mathrm{F}}$, $\mathbf{C}_{\mathrm{F}}$, and $\mathbf{K}_{\mathrm{F}}$ the mass, damping and stiffness [r x r]-matrices of the foundation itself, respectively.

The dynamic stiffness matrix $\mathbf{K}_{\mathrm{dyn}}(\omega)$ is determined in order to take into account the effects of the soil, such as the soil-foundation interaction (SFI), the foundation-soil-foundation interaction (FSFI), the hysteretic damping as well as the radiation or geometric damping without resorting to a large finite element model of the soil. The dynamic impedance matrix is computed by condensing out the entire soil-foundations system onto the foundation interfaces in the frequency domain. It relates the displacements in the nodes on the structure-soil interface to the interaction forces $\mathbf{f}_{\mathrm{s}}(\omega)$ of the unbounded soil. Both dynamic impedance matrix $\mathbf{K}_{\mathrm{dyn}}(\omega)$ and the interaction force vector $\mathbf{f}_{\mathrm{s}}(\omega)$ are obtained from linear elastodynamic problems solved by means of Boundary Element Method (BEM) approach. BEM is based in the validity of the superposition principle and hence, it is conveniently formulated in the frequency domain where hysteretic damping is rigorously valid because of its non-causal nature (Crandall 1970).

The vector $\mathbf{f}_{\mathrm{SF}}(\omega)$ collects the loads at the soil-foundation interface due to the free-field motion as follows:

$$\mathbf{f}_{SF}(\omega) = \mathbf{f}_{s}(\omega)u_{g}(\omega) \qquad (4)$$

where $\mathbf{f}_{s}(\omega)$ is the [r x 1] seismic force vector calculated at the interface for an unit harmonic displacement by means of the BEM analysis and $u_{g}(\omega)$ is the free field motion displacement at the ground surface.

3 Determination of the reduced model

In this paper, the Craig-Bampton (Bampton and Craig 1968) reduction method for modal substructuring is applied in order to determine the reduced model used to perform efficient time-domain sensitivity analysis. The first step involves the conversion of both impedance matrix, $\mathbf{K}_{\text{dyn}}(\omega)$, and the interaction forces $\mathbf{f}_{\text{s}}(\omega)$ from the frequency into the time domain by adopting the lumped-parameter model (LPM) approach. The LPM is constituted by a combination of frequency - independent springs, dashpots and masses opportunely calibrated in order to simulate the dynamic behavior in the frequency domain consistent to that obtained from the BEM analysis. The calibration of the parameter of the LPM contained in the matrices $\mathbf{K}_{\text{LPM}}$, $\mathbf{M}_{\text{LPM}}$, and $\mathbf{C}_{\text{LPM}}$, is based on the approximation of each frequency-dependent component of the impedance matrix $\mathbf{K}_{\text{dyn}}(\omega)$ by means of a least-square regression procedure as follows:

$$\mathrm{K}_{\text{dyn}}^{\text{j,k}}(\omega) \cong \mathrm{K}_{\text{LPM}}^{\text{j,k}} - \omega^2 \mathrm{M}_{\text{LPM}}^{\text{j,k}} + i\omega \mathrm{C}_{\text{LPM}}^{\text{j,k}} \; (\text{j}, \text{k} = 1, \dots, \text{r}) \tag{5}$$

in a given frequency range $0 < \omega < \omega_{\text{cut\_off}}$. Therefore, by using the frequency-independent matrices of Eq. (5), Eq. (1) can be converted in the time domain as follows:

$$\mathbf{M}\ddot{\mathbf{u}}(\text{t}) + \mathbf{C}\dot{\mathbf{u}}(\text{t}) + \mathbf{K}\mathbf{u}(\text{t}) = \mathbf{f}(\text{t}) \tag{6}$$

where $\ddot{\mathbf{u}}(\text{t})$, $\dot{\mathbf{u}}(\text{t})$, and $\mathbf{u}(\text{t})$ corresponds to the [n x 1] vectors of the nodal absolute accelerations, velocities and displacements as functions of time t, and $\mathbf{f}(\text{t})$ is the [n x 1] vector of nodal time-varying applied forces derived by the inverse Fourier transform of the force vector $\mathbf{f}(\omega)$ of Eq. (1).

The mass, damping and stiffness matrix namely, **M**, **C**, and **K** are the approximated matrices of their corresponding $\mathbf{M}_{\text{glob}}$, $\mathbf{C}_{\text{glob}}$, and $\mathbf{K}_{\text{glob}}$ of Eq. (2) where the following relations are used:

$$\begin{aligned} \mathbf{M}_{\text{SF}} &= \mathbf{M}_{\text{F}} + \mathbf{M}_{\text{LPM}} \\ \mathbf{C}_{\text{SF}} &= \mathbf{C}_{\text{F}} + \frac{\text{Im}\{\mathbf{K}_{\text{dyn}}(\omega)\}}{\omega} \cong \mathbf{C}_{\text{F}} + \mathbf{C}_{\text{LPM}} \\ \mathbf{K}_{\text{SF}} &= \mathbf{K}_{\text{F}} + \text{Re}\{\mathbf{K}_{\text{dyn}}(\omega)\} \cong \mathbf{K}_{\text{F}} + \mathbf{K}_{\text{LPM}} \end{aligned} \tag{7}$$

$\mathbf{K}_{\text{F}}$, $\mathbf{M}_{\text{F}}$ and $\mathbf{C}_{\text{F}}$ are the stiffness, the mass and the viscous damping [r x r]-matrices of the foundation itself. Eq. (6) can be rewritten in the following expression:

$$\begin{bmatrix} \mathbf{M}_{\text{ViBa}} & \mathbf{0} & \mathbf{M}_{\text{ViBa,SF}} \\ \mathbf{0} & \mathbf{M}_{\text{str}} & \mathbf{M}_{\text{str,SF}} \\ \mathbf{M}_{\text{SF,ViBa}} & \mathbf{M}_{\text{SF,str}} & \mathbf{M}_{\text{F}} + \mathbf{M}_{\text{LPM}} \end{bmatrix} \ddot{\mathbf{u}}(\text{t}) + \begin{bmatrix} \mathbf{C}_{\text{ViBa}} & \mathbf{0} & \mathbf{C}_{\text{ViBa,SF}} \\ \mathbf{0} & \mathbf{C}_{\text{str}} & \mathbf{C}_{\text{str,SF}} \\ \mathbf{C}_{\text{SF,ViBa}} & \mathbf{C}_{\text{SF,str}} & \mathbf{C}_{\text{F}} + \mathbf{C}_{\text{LPM}} \end{bmatrix} \dot{\mathbf{u}}(\text{t}) + \begin{bmatrix} \mathbf{K}_{\text{ViBa}} & \mathbf{0} & \mathbf{K}_{\text{ViBa,SF}} \\ \mathbf{0} & \mathbf{K}_{\text{str}} & \mathbf{K}_{\text{str,SF}} \\ \mathbf{K}_{\text{SF,ViBa}} & \mathbf{K}_{\text{SF,str}} & \mathbf{K}_{\text{F}} + \mathbf{K}_{\text{LPM}} \end{bmatrix} \mathbf{u}(\text{t}) = \mathbf{f}(\text{t}) \tag{8}$$

Afterwards, the Craig-Bampton method is applied. The method consists of partitioning the global system into two or more subdomains by holding the boundary conditions fixed and then combining the fixed base modal shapes with the constraint modes of the common interface by means of a modal synthesis. Hereafter, the formulation is specialized to the specific case involved in the paper and depicted in Figure 1 even though it can be easily generalized to include more structures and ViBa devices. The physical coordinates $\mathbf{u}$, are transformed to a hybrid set of physical coordinates at the boundary $\mathbf{u}_{\mathrm{SF}}$, and modal coordinates at the interior points of the structure, $\mathbf{q}_{\mathrm{str}}$, and of the ViBa, $\mathbf{q}_{\mathrm{ViBa}}$. By truncating the modal coordinates to smaller sets, let us consider $\boldsymbol{\psi}_{[\mathrm{pxi}]}^{\mathrm{ViBa}}$ and $\boldsymbol{\psi}_{[\mathrm{qxl}]}^{\mathrm{str}}$ as the [p x i] and [q x l]-matrices of the dynamic modal shapes obtained by conventional eigenvalues problem; $\boldsymbol{\phi}_{[\mathrm{pxr}]}^{\mathrm{ViBa}}$ and $\boldsymbol{\phi}_{[\mathrm{qxr}]}^{\mathrm{str}}$ as the [p x r] and [q x r] matrices of interface modal shapes of ViBa and structure, respectively. The constraint modes or interface modes $\boldsymbol{\phi}$ relate the rigid body static unit displacements at the interface $\mathbf{u}_{\mathrm{SF}}$ to the physical displacements of the elastic degrees of freedom $\mathbf{u}$. Furthermore, in case of rigid foundation, the number of constraint modes contained in $\boldsymbol{\phi}$ is sensibly reduced according to the number of degree of freedoms of the foundation master nodes. Therefore, the generalized coordinate [m= i+l+r]-vector $\mathbf{q}^{\mathrm{T}}$ listed as (the superscript T indicates the transpose operator):

$$\mathbf{q}^{\mathrm{T}} = [\mathbf{q}_{\mathrm{ViBa}}\ \mathbf{q}_{\mathrm{str}}\ \mathbf{u}_{\mathrm{SF}}\ ] \qquad (9)$$

is related to the physical coordinates $\mathbf{u}$, by means of the following relation:

$$\begin{bmatrix} \mathbf{u}_{\mathrm{ViBa}} \\ \mathbf{u}_{\mathrm{str}} \\ \mathbf{u}_{\mathrm{SF}} \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{q}_{\mathrm{ViBa}} \\ \mathbf{q}_{\mathrm{str}} \\ \mathbf{u}_{\mathrm{SF}} \end{bmatrix} \qquad (10)$$

where $\mathbf{P}$ is the reduced Craig-Bampton transformation matrix:

$$\mathbf{P}_{[\mathrm{nxm}]} = \begin{bmatrix} \boldsymbol{\psi}_{[\mathrm{pxi}]}^{\mathrm{ViBa}} & \mathbf{0}_{[\mathrm{pxl}]} & \boldsymbol{\phi}_{[\mathrm{pxr}]}^{\mathrm{ViBa}} \\ \mathbf{0}_{[\mathrm{qxi}]} & \boldsymbol{\psi}_{[\mathrm{qxl}]}^{\mathrm{str}} & \boldsymbol{\phi}_{[\mathrm{qxr}]}^{\mathrm{str}} \\ \mathbf{0}_{[\mathrm{rxi}]} & \mathbf{0}_{[\mathrm{rxl}]} & \mathbf{I}_{[\mathrm{rxr}]} \end{bmatrix} \qquad (11)$$

Note that the physical displacements of the interior points are computed by

$$\begin{cases} \mathbf{u}_{\mathrm{ViBa}} = \boldsymbol{\psi}^{\mathrm{ViBa}}\mathbf{q}_{\mathrm{ViBa}} + \boldsymbol{\phi}^{\mathrm{ViBa}}\mathbf{u}_{\mathrm{SF}} \\ \mathbf{u}_{\mathrm{str}} = \boldsymbol{\psi}^{\mathrm{str}}\mathbf{q}_{\mathrm{str}} + \boldsymbol{\phi}^{\mathrm{str}}\mathbf{u}_{\mathrm{SF}} \end{cases} \qquad (12)$$

The projection of the dynamic governing Eq. (7) over the base $\mathbf{P}$, yields to the Craig-Bampton equation of motion of the reduced model in the time domain:

$$\mathbf{P}^{\mathrm{T}}\mathbf{MP}\ddot{\mathbf{q}}(\mathrm{t}) + \mathbf{P}^{\mathrm{T}}\mathbf{CP}\dot{\mathbf{q}}(\mathrm{t}) + \mathbf{P}^{\mathrm{T}}\mathbf{KP}\mathbf{q}(\mathrm{t}) = \mathbf{P}^{\mathrm{T}}\mathbf{f}(\mathrm{t}) \qquad (13)$$

where the size of each reduced matrices $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$, $\mathbf{P}^{\mathrm{T}}\mathbf{CP}$, and $\mathbf{P}^{\mathrm{T}}\mathbf{KP}$ is [m x m] with m << n.

Furthermore, in case of rigid foundation, the number of constraint modes is sensibly reduced to r = 12 corresponding to the degree of freedoms of the master nodes of the two foundations involved in this paper. Remarkably, it has to be emphasized that the use of frequency-independent LPM allows the transformation of hysteretic damping model for the soil strictly valid only in the frequency domain to a viscous damping model used in the conventional time-history analysis.

4 Dynamic response sensitivity for deterministic load

In this section, the sensitivity of the reduced model obtained by projection of the global system onto the Craig-Bampton base is investigated. The first-order sensitivity of the deterministic response of the system is evaluated by means of the approach presented in Cacciola et al. (2005). The standard sensitivity analysis entails the evaluation of the derivative of the response of the system with respect to significant system parameters, collected in the vector $\boldsymbol{\alpha}$. According to the dynamic modification approach, the significant system parameters are defined in the neighbourhood of prefixed values, called nominal parameter values. Therefore, by denoting with $\boldsymbol{\alpha}_0$ the vector of the nominal parameters, the vector $\boldsymbol{\alpha}$ of the actual values is estimated as $\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 + \boldsymbol{\Delta\alpha}$, where $\boldsymbol{\Delta\alpha}$ lists the small parameter variations from the nominal values. Due to the dependence of the system on the actual values of $\boldsymbol{\alpha}$, let the vector of state variables $\mathbf{z}(\boldsymbol{\alpha}, \mathrm{t})$ of order [2m x 1] be introduced in the form:

$$\mathbf{z}(\boldsymbol{\alpha}, \mathrm{t}) = \begin{bmatrix} \mathbf{q}(\boldsymbol{\alpha}, \mathrm{t}) \\ \dot{\mathbf{q}}(\boldsymbol{\alpha}, \mathrm{t}) \end{bmatrix} \tag{14}$$

Therefore, from Eq. (13), the governing equations of motion in state variable modal space for the reduced model, are derived as follows:

$$\dot{\mathbf{z}}(\boldsymbol{\alpha}, \mathrm{t}) = \mathbf{D}(\boldsymbol{\alpha})\mathbf{z}(\boldsymbol{\alpha}, \mathrm{t}) + \mathbf{V}(\boldsymbol{\alpha})\mathbf{f}(\mathrm{t}) \tag{15}$$

where:

$$\mathbf{D}(\boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{0}_{[\mathrm{mxm}]} & \mathbf{I}_{[\mathrm{mxm}]} \\ -\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{K}(\boldsymbol{\alpha})\mathbf{P}_{\boldsymbol{\alpha}_0} & -\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{C}(\boldsymbol{\alpha})\mathbf{P}_{\boldsymbol{\alpha}_0} \end{bmatrix} \tag{16}$$

with $\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}} = \mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{M}(\boldsymbol{\alpha})\mathbf{P}_{\boldsymbol{\alpha}_0}$, and

$$\mathbf{V}(\boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{0}_{[\mathrm{mxm}]} \\ \widetilde{\mathbf{M}}_{\boldsymbol{\alpha}}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}} \end{bmatrix} \tag{17}$$

The reduced Craig-Bampton transformation matrix $\mathbf{P}_{\boldsymbol{\alpha}_0}$ is evaluated according to Eq. (11) in correspondence of the nominal values $\boldsymbol{\alpha}_0$. Therefore, by differentiating Eq. (15) with respect to ith significant parameter α_i , the evolution of the deterministic sensitivity in the neighbourhood of nominal values $\boldsymbol{\alpha}_0$**,** is governed by the following first-order differential equations:

$$\dot{\mathbf{s}}_{\mathrm{z,i}}(\boldsymbol{\alpha}_0, \mathrm{t}) = \mathbf{D}(\boldsymbol{\alpha}_0)\mathbf{s}_{\mathrm{z,i}}(\boldsymbol{\alpha}_0, \mathrm{t}) + \mathbf{A}_\mathrm{i}(\boldsymbol{\alpha}_0)\mathbf{z}(\boldsymbol{\alpha}_0, \mathrm{t}) + \mathbf{B}_\mathrm{i}(\boldsymbol{\alpha}_0)\mathbf{f}(\mathrm{t}) \tag{18}$$

where by denoting with $[\cdot]^{\mathrm{I}}$ the derivative of the matrices with respect to ith significant parameter α_i:

$$\mathbf{A}_\mathrm{i}(\boldsymbol{\alpha}_0) = \frac{\partial}{\partial \alpha_\mathrm{i}}\mathbf{D}(\boldsymbol{\alpha})\Big|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{a}_{1,\mathrm{i}}(\boldsymbol{\alpha}_0) & -\mathbf{a}_{2,\mathrm{i}}(\boldsymbol{\alpha}_0) \end{bmatrix} \tag{19}$$

in which the quantity $\mathbf{a}_{1,\mathrm{i}}(\boldsymbol{\alpha}_0)$ is

$$\mathbf{a}_{1,\mathrm{i}} = -\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0,\mathrm{i}}^{\mathrm{I}}\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{K}\mathbf{P}_{\boldsymbol{\alpha}_0} + \widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{K}_\mathrm{i}^{\mathrm{I}}\mathbf{P}_{\boldsymbol{\alpha}_0} \tag{20}$$

and the quantity $\mathbf{a}_{2,\mathrm{i}}(\boldsymbol{\alpha}_0)$ is

$$\mathbf{a}_{2,\mathrm{i}} = -\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0,\mathrm{i}}^{\mathrm{I}}\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{C}\mathbf{P}_{\boldsymbol{\alpha}_0} + +\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{C}_{\mathrm{i}}^{\mathrm{I}}\mathbf{P}_{\boldsymbol{\alpha}_0} \tag{21}$$

In the above equations, the following position $\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0} = \mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}}\mathbf{M}(\boldsymbol{\alpha}_0)\mathbf{P}_{\boldsymbol{\alpha}_0}$ is used and the arguments are omitted. Finally, $\mathbf{B}_{\mathrm{i}}(\boldsymbol{\alpha}_0)$ is:

$$\mathbf{B}_{\mathrm{i}}(\boldsymbol{\alpha}_0) = \begin{bmatrix} \mathbf{0}_{[\mathrm{mxm}]} \\ -\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0,\mathrm{i}}^{\mathrm{I}}\widetilde{\mathbf{M}}_{\boldsymbol{\alpha}_0}^{-1}\mathbf{P}_{\boldsymbol{\alpha}_0}^{\mathrm{T}} \end{bmatrix} \tag{22}$$

Therefore, the sensitivity of the response of the non-classically damped system is obtained by numerically integrating the sensitivity equation of Eq. (18) by means of the same numerical procedure adopted to solve the governing equation of the motion Eq. (15) due to their similarity between them once the pseudo-force is calculated as:

$$\bar{\mathbf{F}}(\boldsymbol{\alpha}_0, t) = \mathbf{A}_{\mathrm{i}}(\boldsymbol{\alpha}_0)\mathbf{z}(\boldsymbol{\alpha}_0, \mathrm{t}) + \mathbf{B}_{\mathrm{i}}(\boldsymbol{\alpha}_0)\mathbf{f}(\mathrm{t}) \tag{23}$$

This method allows the evaluation of the sensitivity response according to the dynamic modification approach in which the Craig-Bampton transformation matrix is determined at the nominal values $\boldsymbol{\alpha}_0$ and not at the exact values $\boldsymbol{\alpha}$. Therefore, the following approximation $\mathbf{P}(\boldsymbol{\alpha}) \cong \mathbf{P}_{\boldsymbol{\alpha}_0}$ is done; the error in the results is small if the modal shapes is not sensibly affected from the parameters $\boldsymbol{\alpha}$.

5 Dynamic response sensitivity for stochastic load process

In this section, the sensitivity of the response for a system subjected to non-stationary zero mean Gaussian stochastic load process is accomplished. Consider the filtered multi-variate process fully defined by the knowledge of its power spectral density matrix:

$$\mathbf{S}_{\mathrm{FF}}(\omega, \mathrm{t}) = \begin{bmatrix} 0 \\ 0 \\ \mathbf{f}_{\mathrm{s}}(\omega) \end{bmatrix} [\mathbf{0} \quad \mathbf{0} \quad \mathbf{f}_{\mathrm{s}}^{*}(\omega)] \mathrm{S}_{\mathrm{g}}(\omega, \mathrm{t}) \tag{24}$$

where $[\cdot]^*$ denotes the conjugate transpose operator, and is power spectral density function $\mathrm{S}_{\mathrm{g}}(\omega, \mathrm{t})$ of the free field ground motion. The sensitivity of the stochastic response is evaluated in terms of variation of the statistical state-space second-order moment; therefore a direct linear stochastic differential equation of motion is determined.

The second-order moment of the response in the state variable space can be obtained as:

$$\mathbf{m}_{\mathbf{Z}}^{(2)}(\boldsymbol{\alpha}, \mathrm{t}) = \mathrm{E}[\mathbf{Z}(\boldsymbol{\alpha}, \mathrm{t}) \otimes \mathbf{Z}(\boldsymbol{\alpha}, \mathrm{t})] = \mathrm{Vec}\{\mathrm{E}[\mathbf{Z}(\boldsymbol{\alpha}, \mathrm{t})\mathbf{Z}^{\mathrm{T}}(\boldsymbol{\alpha}, \mathrm{t})]\} \tag{25}$$

where $\mathrm{E}[\cdot]$ denotes mathematical expectation, the symbol $\otimes$ is the Kronecker product and $\mathbf{m}_{\mathbf{Z}}^{(2)}(\boldsymbol{\alpha}, \mathrm{t})$ is the vectorialized form, $\mathrm{Vec}\{\cdot\}$, of the cross-covariance matrix defined for the dynamically modified system.

By differentiating Eq. (25) with respect the time, the evolution of the second-order statistical moments of the response in the state variable space is determined after simple algebra:

$$\dot{\mathbf{m}}_{\mathbf{Z}}^{(2)}(\boldsymbol{\alpha}, \mathrm{t}) = \mathbf{D}_{\mathbf{2}}(\boldsymbol{\alpha})\mathbf{m}_{\mathbf{Z}}^{(2)}(\boldsymbol{\alpha}, \mathrm{t}) + \mathbf{F}_{2}(\boldsymbol{\alpha}, \mathrm{t}) \tag{26}$$

where by using the Kronecker sum $\oplus$:

$$\mathbf{F}_2(\boldsymbol{\alpha},\mathrm{t}) = [\mathbf{I}_{2\mathrm{m}}\otimes\mathbf{V}(\boldsymbol{\alpha})]\mathrm{E}[\mathbf{Z}(\boldsymbol{\alpha},\mathrm{t})\otimes\mathbf{f}(\mathrm{t})] + [\mathbf{V}(\boldsymbol{\alpha})\otimes\mathbf{I}_{2\mathrm{m}}]\mathrm{E}[\mathbf{f}(\mathrm{t})\otimes\mathbf{Z}(\boldsymbol{\alpha},\mathrm{t})] \quad (27)$$

in which $\mathbf{I}_{2\mathrm{m}} = \mathbf{I}_{[2\mathrm{m}\mathrm{x}2\mathrm{m}]}$ and :

$$\mathbf{D}_2(\boldsymbol{\alpha}) = \mathbf{D}(\boldsymbol{\alpha})\oplus\mathbf{D}(\boldsymbol{\alpha}) \qquad (28)$$

Finally, the differential equations governing the evolution of the sensitivity of the second-order statistical moments of the response in the state variable space is determined by differentiating Eq. (26) with respect to ith significant parameter α_{i}:

$$\dot{\mathbf{s}}_{\mathbf{Z},\mathrm{i}}^{(2)}(\boldsymbol{\alpha}_0,\mathrm{t}) = \mathbf{D}_2(\boldsymbol{\alpha}_0)\mathbf{s}_{\mathbf{Z},\mathrm{i}}^{(2)}(\boldsymbol{\alpha}_0,\mathrm{t}) + \bar{\mathbf{F}}_2(\boldsymbol{\alpha}_0,\mathrm{t}) \qquad (29)$$

where the pseudo-force $\bar{\mathbf{F}}_2(\boldsymbol{\alpha}_0,\mathrm{t})$ lists the cross-correlation terms as follows:

$$\begin{aligned}\bar{\mathbf{F}}_2(\boldsymbol{\alpha}_0,\mathrm{t}) &= \mathbf{A}_{2,\mathrm{i}}(\boldsymbol{\alpha}_0)\mathbf{m}_{\mathbf{Z}}^{(2)}(\boldsymbol{\alpha}_0,\mathrm{t}) \\ &+[\mathbf{I}_{2\mathrm{m}}\otimes\mathbf{V}(\boldsymbol{\alpha}_0)]\mathrm{E}\left[\mathbf{s}_{\mathrm{z},\mathrm{i}}(\boldsymbol{\alpha}_0,\mathrm{t})\otimes\mathbf{f}(\mathrm{t})\right] + [\mathbf{V}(\boldsymbol{\alpha}_0)\otimes\mathbf{I}_{2\mathrm{m}}]\mathrm{E}\left[\mathbf{f}(\mathrm{t})\otimes\mathbf{s}_{\mathrm{z},\mathrm{i}}(\boldsymbol{\alpha}_0,\mathrm{t})\right] \\ &+ [\mathbf{I}_{2\mathrm{m}}\otimes\mathbf{B}(\boldsymbol{\alpha}_0)]\mathrm{E}[\mathbf{Z}(\boldsymbol{\alpha}_0,\mathrm{t})\otimes\mathbf{f}(\mathrm{t})] \\ &+ [\mathbf{B}(\boldsymbol{\alpha}_0)\otimes\mathbf{I}_{2\mathrm{m}}]\mathrm{E}[\mathbf{f}(\mathrm{t})\otimes\mathbf{Z}(\boldsymbol{\alpha}_0,\mathrm{t})]\end{aligned} \qquad (30)$$

with

$$\mathbf{A}_{2,\mathrm{i}}(\boldsymbol{\alpha}_0) = \mathbf{A}_{\mathrm{i}}(\boldsymbol{\alpha}_0)\oplus\mathbf{A}_{\mathrm{i}}(\boldsymbol{\alpha}_0) \qquad (31)$$

Consider the input as non-stationary zero-mean Gaussian stochastic process fully defined by the power spectral density (PSD) matrix function $\mathbf{S}_{\mathrm{FF}}(\omega,\mathrm{t})$ as defined in Eq. (24). After simple algebra, the stochastic averages $\mathrm{E}[\mathbf{Z}(\boldsymbol{\alpha},\mathrm{t})\otimes\mathbf{f}(\mathrm{t})]$ and $\mathrm{E}[\mathbf{f}(\mathrm{t})\otimes\mathbf{Z}(\boldsymbol{\alpha},\mathrm{t})]$ of Eq. (30) are obtained as follows:

$$\mathrm{E}[\mathbf{Z}(\boldsymbol{\alpha},\mathrm{t})\otimes\mathbf{f}(\mathrm{t})] \cong \int_{-\infty}^{\infty}[\mathbf{H}_0^{*}(\omega,\boldsymbol{\alpha})\otimes\mathbf{I}_{\mathrm{m}}]\,\mathrm{Vec}\{\mathbf{S}_{\mathrm{FF}}(\omega,\mathrm{t})\}\mathrm{d}\omega \qquad (32)$$

and

$$\mathrm{E}[\mathbf{f}(\mathrm{t})\otimes\mathbf{Z}(\boldsymbol{\alpha},\mathrm{t})] \cong \int_{-\infty}^{\infty}[\mathbf{I}_{\mathrm{m}}\otimes\mathbf{H}_0(\omega,\boldsymbol{\alpha})]\,\mathrm{Vec}\{\mathbf{S}_{\mathrm{FF}}(\omega,\mathrm{t})\}\mathrm{d}\omega \qquad (33)$$

in which $\mathbf{I}_{\mathrm{m}} = \mathbf{I}_{[\mathrm{m}\mathrm{x}\mathrm{m}]}$. The matrix $\mathbf{H}_0(\omega,\boldsymbol{\alpha})$ is the transfer function matrix of the system in the frequency domain:

$$\mathbf{H}_0(\omega,\boldsymbol{\alpha}) = [\mathrm{i}\omega\mathbf{I}_{2\mathrm{m}} - \mathbf{D}(\boldsymbol{\alpha})]^{-1}\mathbf{V}(\boldsymbol{\alpha}) \qquad (34)$$

It should be emphasized that the relations in Eq. (32) and in Eq. (33) are exact only in case of stationary input since it is assumed that the transfer function $\mathbf{H}_0(\omega,\boldsymbol{\alpha})$ is independent with respect to the time.

By differentiating Eq. (32) and Eq. (33) with respect to the sensitivity parameter $\boldsymbol{\alpha}$, the cross-correlation functions $\mathrm{E}\left[\mathbf{s}_{z,i}(\boldsymbol{\alpha},t)\otimes\mathbf{f}(t)\right]$ and $\mathrm{E}\left[\mathbf{f}(t)\otimes\mathbf{s}_{z,i}(\boldsymbol{\alpha},t)\right]$ are derived as follows:

$$\mathrm{E}\left[\mathbf{s}_{z,i}(\boldsymbol{\alpha},t)\otimes\mathbf{f}(t)\right]=\int_{-\infty}^{\infty}\left\{\{[\mathbf{H}_0^*(\omega,\boldsymbol{\alpha})]^{-1}\mathbf{A}_i(\boldsymbol{\alpha})[\mathbf{H}_0^*(\omega,\boldsymbol{\alpha})]^{-1}+[\mathbf{H}_0^*(\omega,\boldsymbol{\alpha})]^{-1}\mathbf{B}(\boldsymbol{\alpha})\}\otimes\mathbf{I}_m\right\}\mathrm{Vec}\{\mathbf{S}_{FF}(\omega,t)\}d\omega \tag{35}$$

and

$$\mathrm{E}\left[\mathbf{f}(t)\otimes\mathbf{s}_{z,i}(\boldsymbol{\alpha},t)\right]=\int_{-\infty}^{\infty}\left\{\mathbf{I}_m\otimes\{[\mathbf{H}_0(\omega,\boldsymbol{\alpha})]^{-1}\mathbf{A}_i(\boldsymbol{\alpha})[\mathbf{H}_0(\omega,\boldsymbol{\alpha})]^{-1}+[\mathbf{H}_0(\omega,\boldsymbol{\alpha})]^{-1}\mathbf{B}(\boldsymbol{\alpha})\}\right\}\mathrm{Vec}\{\mathbf{S}_{FF}(\omega,t)\}d\omega \tag{36}$$

By calculating each cross-correlation term, the pseudo-force of Eq. (30) is derived and the sensitivity of the response for a system subjected to zero mean Gaussian quasi-stationary stochastic load process is accomplished.

6 Numerical Application

In this section, the proposed procedure is applied to investigate the sensitivity of stochastic response of the model of an Industrial Building protected by the ViBa as depicted in Figure 2. The simple model of a Reactor Building, described in the report EPRI (2006), is chosen. The relevant dimensions are summarized in Table 1. The model of Reactor Building is founded on 30m-thick soil deposit characterized by shear wave velocity $V_s = 400\, m/s^2$and hysteretic damping $\eta_s = 0.1$ resting on stiff bedrock with shear wave velocity of $V_s = 800\, m/s^2$ and hysteretic damping $\eta_{bed} = 0.05$. The ViBa is externally modelled as a circular embedded foundation characterized by dimensions of the radius and the embedded height equal to half as much as those related to the Reactor Building as reported in Table 2. Both structures are modelled by concrete shell elements. The internal structure of the ViBa is a single oscillator characterized by the internal mass, m_{ViBa}, the stiffness, k_{ViBa}, and the damping ratio ξ_{ViBa}. The mass m_{ViBa} of the ViBa is initially assigned as 3.1×10^7 kg corresponding to the 55% of the mass of the Reactor Building. The model of the Reactor Building is modelled according to the finite element approach by means of the Code_Aster open source FE-software (2013) whereas the BEM formulation is used to model the soil by means of Miss3D (Clouteau, 2005). The sensitivity of the stochastic response of the Industrial Building is investigated by means of the procedure proposed in this paper.

Firstly, a reduced model is obtained in order to capture the structural behavior that is aimed to be protected, lastly, the stochastic sensitivity of the reduced model with respect to the ViBa parameters is evaluated. The pertinent vector listing the ViBa sensitivity parameters is chosen as:
$\boldsymbol{\alpha} = [k_{ViBa}, \quad \xi_{ViBa}]$
while the vector of nominal parameter is defined here as:

$$\boldsymbol{\alpha}_0 = [1.8634\mathrm{E}+10\; N/m \quad 0.01] \tag{37}$$

According to the proposed procedure, the reduced global model is obtained once the vector of generalized coordinates $\mathbf{q}^T = [q^x_{ViBa}\,,\, q^x_{str}, u^x_{SF,str}, u^\theta_{SF,str}, u^x_{SF,ViBa}, u^\theta_{SF,ViBa}]$ is determined, where

q^x_{ViBa} and q^x_{str} are the first generalized coordinates in x-direction of the ViBa and structure, respectively;

Table 1 Significant dimensions of the Reactor Building

Reactor Building shell radius	25.8 m
Basement shell radius	25.8 m
Height of springline above basemat	46.12 m
Embedded height	12.9 m
Wall thickness	1.07 m
Basemat thickness	3.05 m

Table 2 Significant dimensions of the proposed device ViBa

Basement shell radius	12.9 m
Distance from Reactor	12.9 m
Embedded height	6.45 m
Wall thickness	1.5 m
Basemat thickness	1.5 m

$u^x_{SF,str}$ $u^\theta_{SF,str}$ $u^x_{SF,ViBa}$ $u^\theta_{SF,ViBa}$ are the x-and θ - directions related to the structure and ViBa, respectively.

In order to formulate the model in the time domain, a lumped parameter model is determined for approximating each component of the dynamic stiffness matrix $\mathbf{K}_{dyn}(\omega)$ by frequency-independent parameters obtained by linear least squares in the range 0-5 Hz. Conversely, the LPM used for the real part of the foundations coupling impedances, indicated by $K^{i,j}_{dyn}(\omega)$ with $i \neq j$, is obtained by calibrating the elastic spring at the first fundamental frequency of the Reactor Building in the coupled case, i.e. 3.65 Hz.

Ground motion acceleration $\ddot{u}_g(t)$ is modelled as a broadband uniformly modulated process $S_g(\omega, t) = \varphi(t)S_w$ with the cut-off frequency of 5 Hz, $S_w = 10^{-6}$ m²/s³ and modulating function $\varphi(t)$ given by:

$$\varphi(t) = 12.21(e^{-0.4t} - e^{-0.5t}) \quad (38)$$

Monte Carlo Simulation (MCS) is performed by generation of one hundred samples from the adopted input process for both the reduced model and the exact reference model.

In Figure 3a-b are reported the curves of evolution of the second-order horizontal modal moment $m^{(2)}_{q_{str}}(t)$ obtained by solving Eq. (26) for both the structure and the ViBa. The curves are related the first two generalized coordinates, q^x_{str} and q^x_{ViBa} of the adopted reduced model. Furthermore, the curves of second-order statistical moments are then compared with the moments $m^{(2)}_u(t)$ calculated by the results obtained by the MCS for the exact reference model in Code_Aster; the comparison shows the good matching of the results indicating the accuracy of the reduced model to simulate the exact reference model.

Finally, Figure 3 shows the evolution of the second-order horizontal modal moment of the structure in case of absence of coupling interaction with the ViBa. It is worth noting the important effect of the ViBa in reducing the second order moment of the structure with respect to the uncoupled case. Then, the sensitivity analysis is performed by the procedure proposed in this paper. The curves of the sensitivity of the second-order state space moment $s_{q,\alpha}^{(2)}(t)$ of the structure with respect to the variation of the ViBa stiffness k_{ViBa} and damping ξ_{ViBa}, are depicted in Figure 4a-b, respectively. Moreover, the results obtained by MCS of the exact reference model are illustrated for comparison purpose. In order to have a better interpretation of the sensitivity of the response, the results are represented in the non-dimensional form as follows:

$$S_{q,\alpha}^{(2)}(t) = \frac{s_{q,\alpha}^{(2)}(t)}{m_q^{(2)}(t)} \alpha_0 \qquad (39)$$

The non-dimensional sensitivity expresses the amplification of the variation of the second-order moment of the response due to a small change in the significant nominal parameter with respect to which the sensitivity is evaluated. Results obtained from Eq. (39) are depicted in Figure 5a-b. As emphasized in Cacciola et al. (2005) the non-dimensional sensitivity can be used to predict the response of the whole system after a small modification in the significant parameters, once the response of nominal system is known; the predicted second-order moment $m_{q,\Delta}^{(2)}(t)$ related to the new nominal value $\boldsymbol{\alpha}_{0,\Delta} = \boldsymbol{\alpha}_0 + \boldsymbol{\Delta\alpha}_0$ is derived by the following formula:

$$m_{q,\Delta}^{(2)}(t) = m_q^{(2)}(t)\left(1 - S_{q,\alpha}^{(2)}(t)\Delta\right) \qquad (40)$$

where Δ is the small variation of the parameter. Figure 6 shows the curves calculated by Eq. (40) for a variation of $\Delta = \pm 10\%$ of the nominal stiffness k_{ViBa} and nominal damping ξ_{ViBa}, respectively. The curves show a good accuracy of Eq. (40) in evaluating the response statistical moments with by the knowledge of their sensitivity, for small variation of a nominal parameter. Moreover, it is worth noting that the change of the response to the variation of stiffness k_{ViBa} as depicted in Figure 6a is noteworthy while the response is less affected by the variation of the damping ξ_{ViBa}, as shown in Figure 6b.

Finally, it should be highlighted that the computational effort is drastically reduced by means of the proposed procedure; the elapsed CPU time calculated on an Intel® Core™ i7-3770 3.40GHz, is 2.97 minutes for the stochastic analysis while the MCS of the exact reference model performed in Code_Aster requires about 313 seconds per record, i.e. around 8.7 hours for 100 samples.

7 Concluding Remarks

The semi-analytical procedure proposed by Cacciola et al. (2005) for evaluating the sensitivity of the reduced model obtained by applying the Craig-Bampton procedure, has been extended in order to take into account for both the alternative base for modal reduction and multi-variate stochastic Gaussian load process. The reduced model provides the drastic reduction of computational effort due to the i) modal truncation of higher modes; ii) the independence of the analysis for each subdomain.

Furthermore, conventional time domain analysis has been performed by resorting to a lumped parameter model derived in order to approximate the frequency-dependent dynamic impedances of the soil with frequency-independent parameters of masses, dashpots and stiffnesses. Therefore, the soil model with hysteretic damping is converted to the approximately equivalent viscous damping model.

The procedure proposed in this paper has been applied to investigate the stochastic response of an Industrial Building coupled with the new proposed device called Vibrating Barrier (ViBa).

The effectiveness of the procedure adopted for the sensitivity analysis and for the approximation of the soil with lumped parameter models has been proved by positive comparison with Monte Carlo Simulation of the exact FE model in evaluating the second-order statistical moments of the stochastic response of the reduced model. Moreover, a relevant reduction of the second-order statistical moment of the Industrial Building has been achieved by the protection provided by the Vibrating Barrier.

Finally, the response sensitivity has been evaluated with respect to the main design parameters of the ViBa.

Very good matching has been achieved between the results obtained by the reduced model and the results of the MCS for the exact FE model performed in Code_Aster. Non-dimensional formulation of the sensitivity has pointed out the importance of the stiffness of the ViBa in evaluating the stochastic response of the Industrial Building.

Acknowledgments

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References

Bampton MCC, Craig, Jr. RR, Coupling of substructures for dynamic analyses, *AIAA Journal.*, 6(7):1313–9 ,Jul;1968.

Cacciola P, Colajanni P, Muscolino G, A modal approach for the evaluation of the response sensitivity of structural systems subjected to non-stationary random processes, *Computer Methods in Applied Mechanics and Engineering*, 194(42-44):4344–61, Oct 2005.

Cacciola P, EPSRC First Grant EP/K004867/1 “Vibrating Barriers for the control of seismic waves (ViBa)”, 2012.

Clouteau D, MISS 6.4. Ecole Centrale de Paris, Chatenay-Malabry, France; 2005.

Crandall, SH, The role of damping in vibration theory, *Journal of Sound and Vibration* 11, 3–IN1, 1970.

EDF R&D, Code_Aster [Internet]. EDF R&D; Available from: http://www.code-aster.org

EPRI, Effect of Seismic Wave Incoherence on Foundation and Building Response. EPRI, Report No.: 1013504, 2006.

Neumark S, Concept of complex stiffness applied to problems of oscillation with viscous and hysteretic damping. *Aero Research Council R&M*, Report No.: 3269, 1957.

Kausel E, Whitman R.V, Morray J.P., Elsabee F, The spring method for embedded foundations. *Nuclear Engineering and Design*, Aug. 1978.

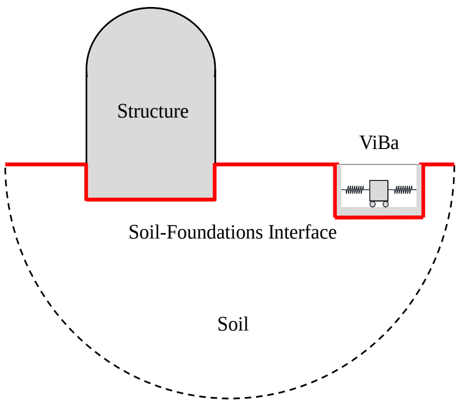

Figure 1 Subdomains of the global problem considered in the paper

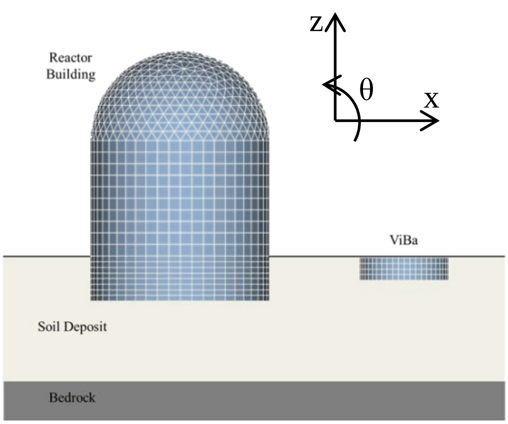

Figure 2 Investigated Case Study of a model of Reactor Building protected by ViBa

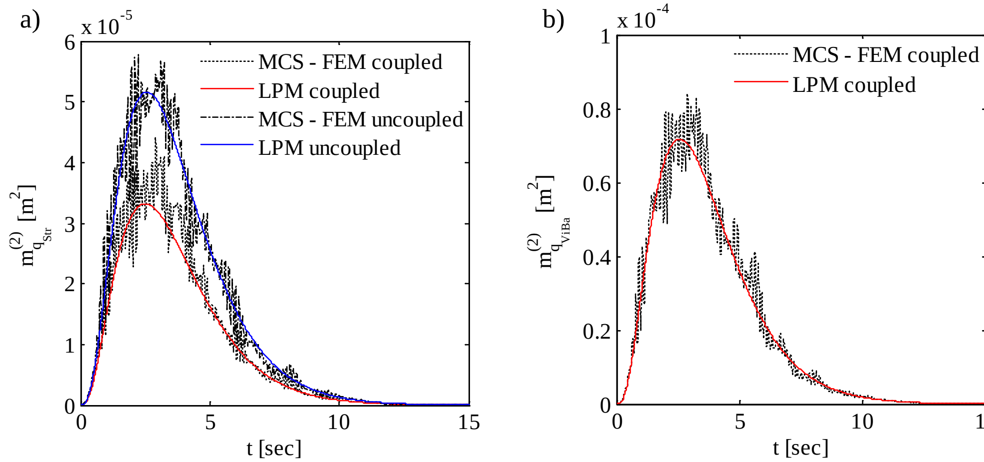

Figure 3 Second-order statistical moments of the modal displacements of the reduced LPM model for a) structure and b) ViBa: comparison with Monte Carlo simulation for the FE model (dotted line).

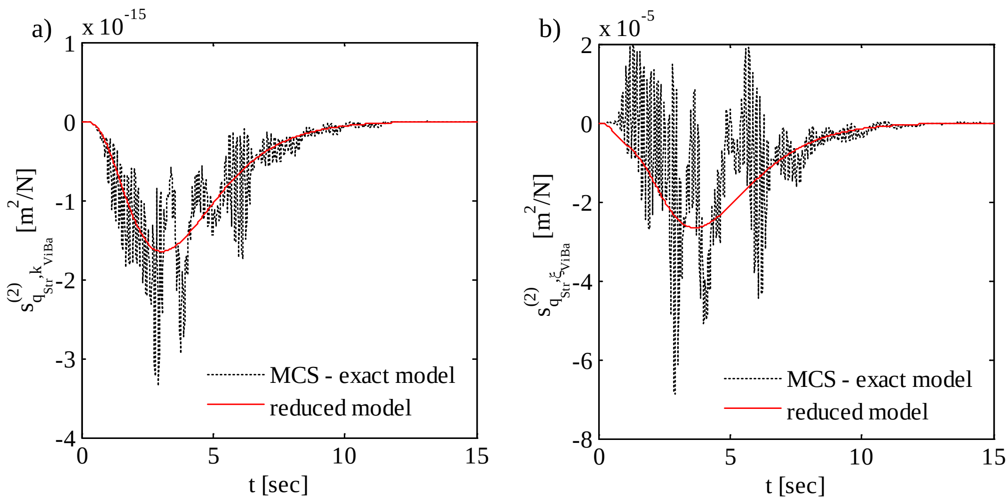

Figure 4 Sensitivity of the second-order statistical moments of the structure with respect to the a) ViBa stiffness and b) ViBa damping

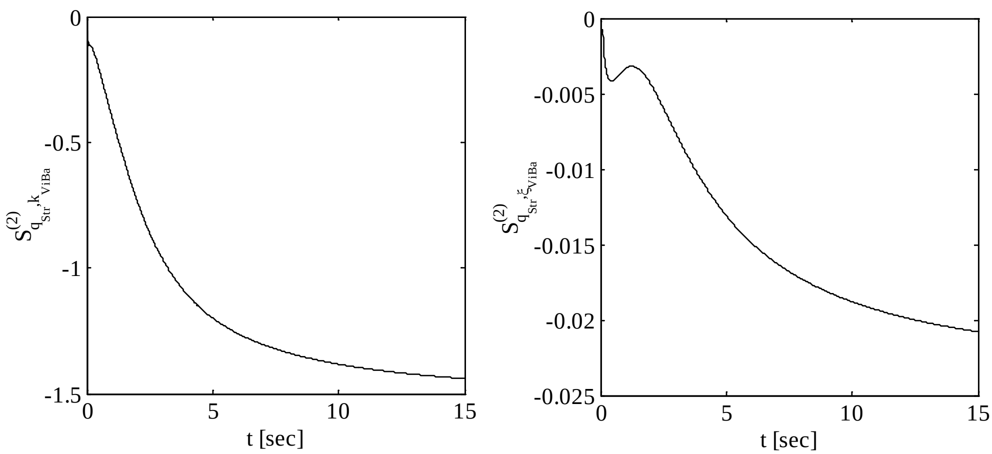

Figure 5 Non-dimensional sensitivity of the second-order statistical moments of the structure with respect to the a) stiffness and the b) damping of the ViBa

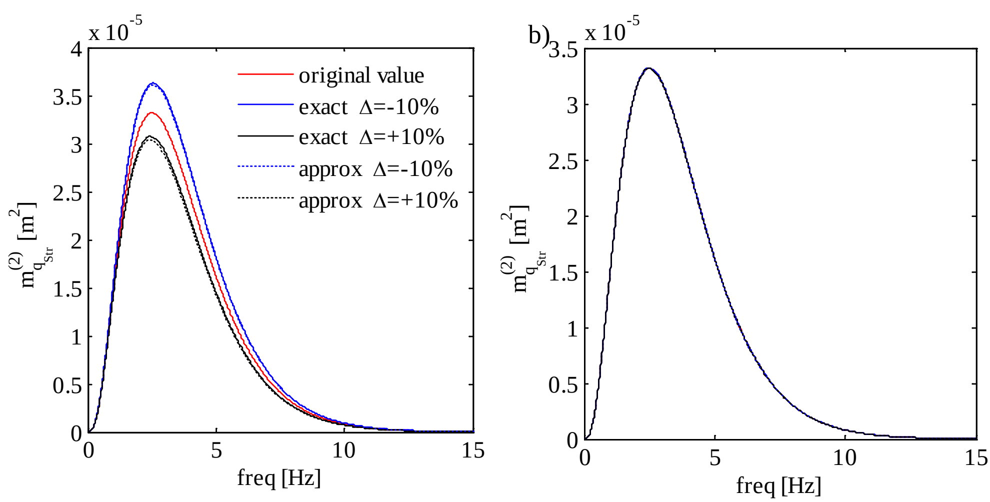

Figure 6 Second-order statistical moments of the modal structural displacements corresponding to a variation of the nominal parameters