

# Analysis of practical engineering approaches to guide the dual scaling of earthquake ground motion for non-linear time-history analysis

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## SUMMARY:

This article introduces two engineering approaches to guide the simultaneous scaling of both amplitude and time of natural accelerograms. This with the objective of providing alternative tools to complement current data sets of earthquake records to be scaled and used for nonlinear time-history analysis. The first approach proposed for dual scaling relies on the explicit application of a ground motion prediction equation (GMPE) for spectral acceleration. The second approach makes use of a pair of GMPEs to estimate ground motion parameters (GMPs) defined in frequency/period and time domains. To characterize the intensity of the earthquake ground motion, the GMPs selected in this study are Arias Intensity and Housner Intensity.

*Keywords: estimation of earthquake ground motion, amplitude-scaling, time-scaling, seismic intensity*

## 1. INTRODUCTION

The intensity of natural accelerograms can be modified by amplitude scaling so as to define the seismic input for nonlinear time-history analysis (*e.g.* Fintel and Gosh, 1982; Kappos, 1991; Kappos and Kyriakakis, 2000, Bazzurro and Luco, 2003; Burati *et al.*, 2011). Amplitude scaling can be even explicitly linked to the fundamental properties of strength and stiffness of the structure under analysis (Martinez-Rueda, 1998), the style of stiffness degradation (Apango-Vera and Martinez-Rueda, 2001) and the type of site conditions where the earthquake ground motion (EGM) occurs (Martinez-Rueda, 2006a). However, amplitude scaling has the limitation of not being able to modify the frequency content of an earthquake record. On the other hand, in many instances it can be difficult to identify a natural accelerogram that leads to critical spectral ordinates associated to the fundamental period of the structure under analysis. Although the effect that time-scaling has on spectral shape has long been recognized, less attention has been given to the potential of time-scaling to artificially generate EGM. Furthermore, to the best of the author's knowledge, a dual scaling procedure that incorporates the simultaneous scaling of both amplitude and time has not been considered in detail yet. As a way to complement existing databases of natural accelerograms this article introduces two engineering approaches that could be incorporated in practice to guide the dual scaling of earthquake records.

### 1.1. Objective and scope

The main objective of this article is to introduce simple engineering approaches to modify by dual scaling a recorded/natural accelerogram to estimate 'new' EGM associated with different earthquake magnitude and/or different distance to the seismic source. The natural accelerogram under scaling is referred to as the *reference earthquake ground motion* (REGM). In principle, the approach for dual scaling introduced in this study is general and hence valid for any type of seismic site (rock, stiff soil, soft soil) and for any GMPEs used to estimate the intensity of the seismic input.

The first approach suggested for dual scaling is fully illustrated by using as an example EGM generated by normal faulting and recorded on rock. Also, the empirical GMPE for spectral acceleration (SA) proposed by Ambraseys *et al.* (2005) to estimate ground in Europe and the Middle East is used to characterise the attenuation of SA. The adopted GMPE for SA is used for illustrative

purposes only; other GMPEs such as that of Akkar and Bommer (2010) or any other from the family of Next Generation Attenuation (NGA) models could be applied within the context of the proposed scaling procedure.

In the interest of clarity, it is important to note that in this article an accelerogram under scaling is visualised as a time-series in which each term of the series consists of a point with time and amplitude coordinates  $(t, \ddot{u}_g(t))$ ; where  $t$  = time and  $\ddot{u}_g(t)$  = ground acceleration. Accordingly, each point of the accelerogram subjected to dual scaling has as modified coordinates  $(SF_t \cdot t, SF_a \cdot \ddot{u}_g(t))$ ; where  $SF_t$  is the time-scaling factor and  $SF_a$  is the amplitude-scaling factor.

## 2. ESTIMATION OF ACCELERATION RESPONSE SPECTRUM

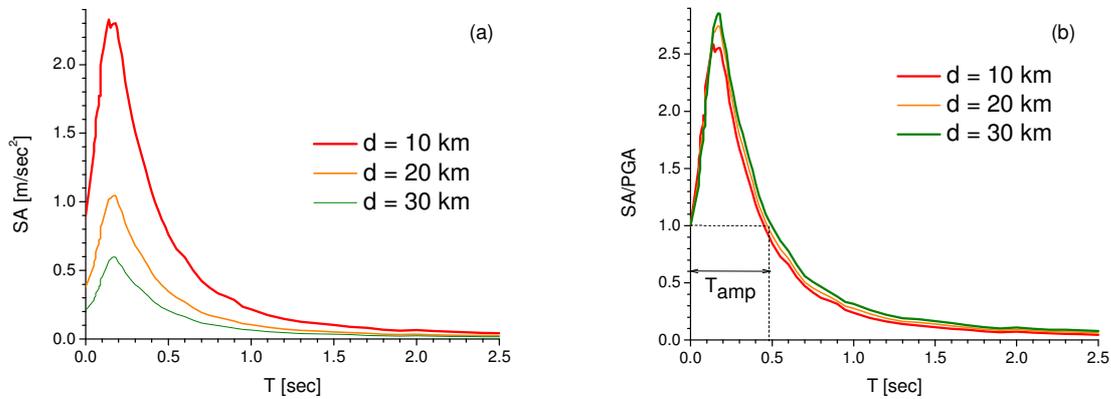
If we consider EGM generated by normal faulting and recorded on rock then the GMPE for SA proposed by Ambraseys *et al.* (2005) is simplified into:

$$\log(SA) = a_1 + a_2 M_w + (a_3 + a_4 M_w) \log \sqrt{d^2 + a_5^2} + a_8 \quad (1)$$

where  $SA$  is the spectral acceleration in  $\text{m/sec}^2$  for a 5% damping ratio;  $M_w$  is the moment magnitude of the earthquake;  $d$  is the distance in km to the surface projection of the fault;  $a_1$  to  $a_8$  are period dependent fitted coefficients given in Ambraseys *et al.* (2005).

### 2.1. Effect of distance to seismic source on the shape of the predicted response spectrum

Figure 1(a) shows a family of mean response spectra predicted by eqn. (1) for the same magnitude, seismic site and style of faulting but different source-to-site distances.

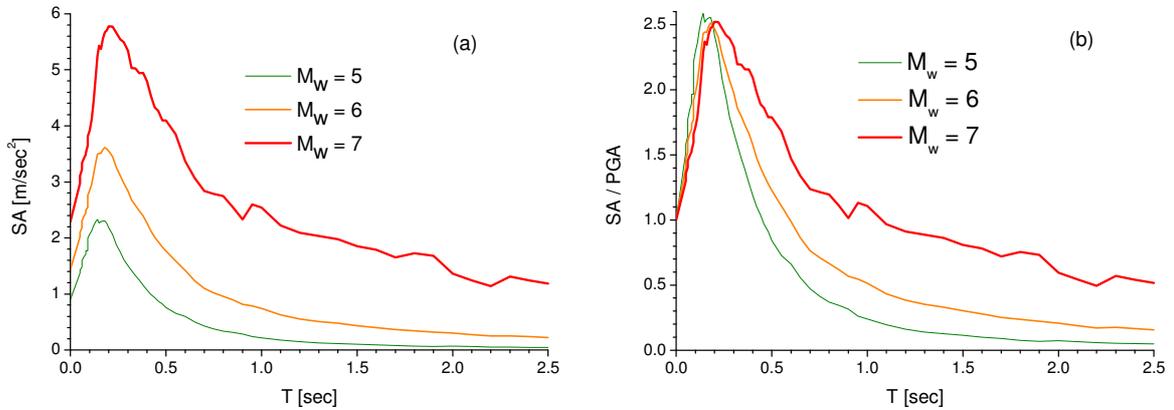


**Figure 1.** Response spectra predicted for  $M_w=5$ , rock and normal faulting; (a) unscaled spectra, (b) normalised spectra.

An alternative visualization of the family of spectra of Figure 1(a) is given in Figure 1(b). This figure clearly shows that if the spectra of Figure 1(a) are normalised with respect to peak ground acceleration ( $PGA$ ) then the normalised spectra exhibit a remarkable similitude. For practical purposes one could even assume that the normalised spectrum shape is virtually the same irrespective of the distance to the source. In consequence, irrespective of  $d$ , the distribution of the frequency content is virtually the same in each of the spectrum of the family. In fact, the period range at which  $SA$  exceeds  $PGA$  remains fairly constant for all the family of spectra. This period range is referred to in this study as the *amplification band*, which is characterised by the period  $T_{amp}$  shown in Figure 1(b). However, one should also keep in mind that the GMPE predicts a ‘mean’ spectral shape and one should expect scatter about the mean of the spectral ordinates. This is just one of the reasons why, to conduct nonlinear inelastic time-history analysis, modern codes require the use of a family of accelerograms rather than a single record.

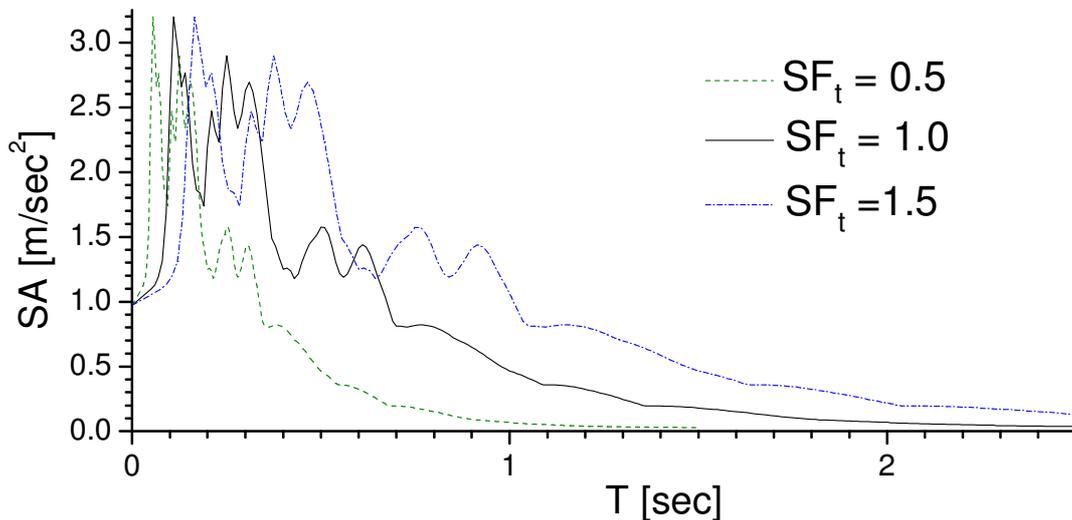
## 2.2. Effect of earthquake magnitude on the shape of the predicted response spectrum

Figure 2(a) shows a family of response spectra generated with eqn. (1) using the same seismic site, style of faulting and source distance  $d$  but different magnitudes.



**Figure 2.** Response spectra predicted for  $d = 10$  km, rock and normal faulting  
(a) unscaled spectra, (b) normalised spectra.

Unlike the spectra of Figure 1(b), Figure 2(b) indicates that if the spectra of Figure 2(a) are normalised with respect to  $PGA$  then it is not realistic to assign a common  $T_{amp}$  for the family of spectra. Nevertheless, it is important to note that the overall shape of the spectra seem to be preserved. In the interest of simplicity, one could argue that, observing the spectra in the normalised space, the main effect of increasing earthquake magnitude (while keeping the other seismologic parameters constant) consists of a widening of the normalised spectrum shape (*i.e.* a widening of the amplification band assessed by  $T_{amp}$ ) with an overall preservation of the relative amplitude of the spectral ordinates. On the other hand, a decrease of earthquake magnitude leads to a narrowing of the normalised spectrum shape.



**Figure 3.** Effect of time-scaling on the spectral shape associated to the accelerogram of Figure 7(a).

The above trends are confirmed in the response spectra of Figure 3. In fact, the overall spectrum shape of the REGM ( $SF_t = 1$ ) is preserved both, when the frequency content is narrowed by using a time-scaling factor  $SF_t = 0.5$  and when the frequency content is widened by using  $SF_t = 1.5$ .

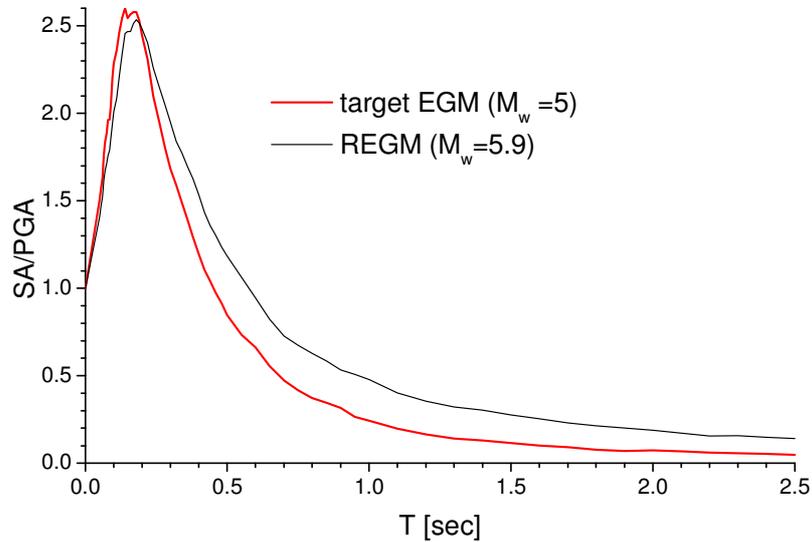
### 3. PROPOSAL OF DUAL SCALING PROCEDURES FOR NATURAL ACCELEROGRAMS

The observations of section 2 are crucial to justify the scaling of accelerograms as a mean to estimate realistic earthquake ground motion. Given a recorded accelerogram with known seismological parameters (*i.e.*  $M_w$ ,  $d$ , seismic site and style of faulting), Figure 1 demonstrates that one can resort to modifying the accelerogram by amplitude-scaling to reflect changes in distance to source. On the other hand, if one needs to reflect a change in magnitude, Figure 2 suggests that a modification of the frequency content is also necessary and hence one can resort to the dual scaling (*i.e.* time-scaling + amplitude-scaling) of the accelerogram. It cannot be overemphasised that, to make sense, the above scaling procedure is only feasible under the umbrella of a GMPE that characterises the intensity and frequency content of the estimated EGM as a function of magnitude and source distance for a given seismic site.

#### 3.1 Dual scaling guided by a GMPE for spectral acceleration

To illustrate the suggested dual scaling procedure an example is presented. The REGM is the accelerogram of Figure 7(a), which was recorded on rock at the station Atina during the  $M_w = 5.9$  earthquake of 07/05/1984 Italian earthquake. According to Ambraseys *et al.* (2005) this station was located at a distance  $d = 11$  km and the EGM was generated by normal faulting. All the spectra of this example referred to as ‘predicted’ are generated using the GMPE of Ambraseys *et al.* (2005), using eqn. (1) with  $d = 11$  km.

Using dual scaling our objective is to find a combination of a time-scaling factor  $SF_t$  and an amplitude-scaling factor  $SF_a$  that modifies the REGM to estimate an accelerogram on rock, generated by normal faulting, occurring at the same distance to source of the REGM but consistent with a reduced magnitude  $M_w = 5$ , which is referred to as the *target magnitude*.

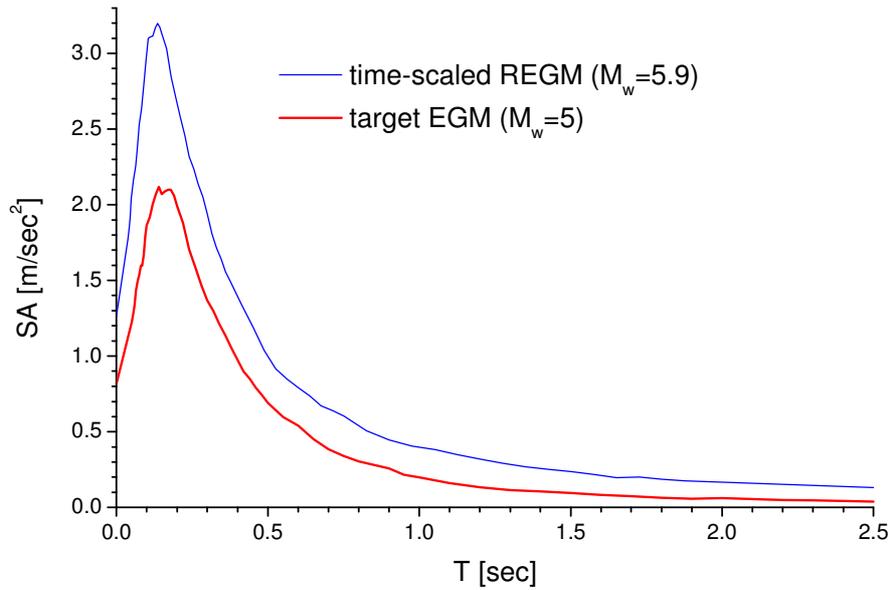


**Figure 4.** Comparison of predicted normalised spectra for reference and target magnitudes.

Figure 4 shows the predicted normalised spectra for  $M_w=5.9$  and  $M_w=5$ . A comparison of the amplification bands of the two spectra allows us to estimate the time-scaling factor as:

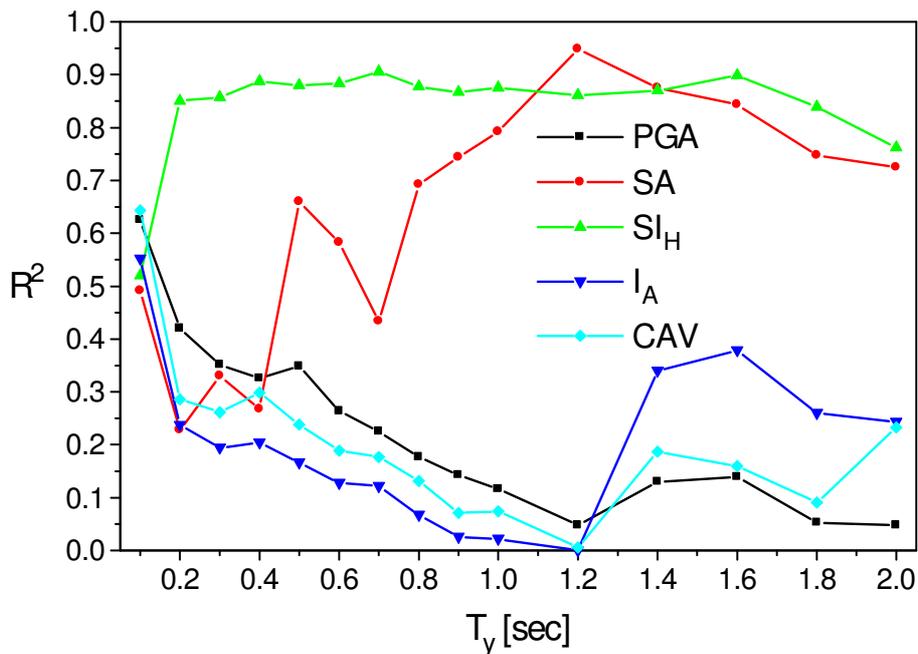
$$SF_t = \frac{T_{amp}(M_w = 5)}{T_{amp}(M_w = 5.9)} = \frac{0.4679 \text{ sec}}{0.6231 \text{ sec}} = 0.7509 \quad (2)$$

To define the amplitude-scaling factor we need to compare the intensity of the EGM associated with the predicted target spectrum ( $M_w=5$ ) and that of the reference record ( $M_w=5.9$ ) affected by time-scaling. The comparison of such spectra is presented in Figure 5.



**Figure 5.** Comparison between the predicted spectrum for the target magnitude and the predicted spectrum, affected by time-scaling, for the reference magnitude.

At first glance we could try to define  $SF_a$  as the point-by-point ratio of the spectral ordinates; however, this ratio is clearly not constant. A better option to define  $SF_a$  is based on the comparison of an overall measure of the EGM intensities involved. In fact, as illustrated in Figure 6, the study of Martinez-Rueda and Vlachos (2010) reveals that the degree of association (assessed by the coefficient of determination  $R^2$ ) between Housner intensity  $SI_H$  and ductility demand is greater and far more stable over a wider range of periods and strengths than that between  $SA$  and ductility demand. Furthermore, Figure 6 indicates that this fact is also true when the effectiveness of  $SI_H$  is compared with other GMPs such as  $PGA$ , Arias Intensity  $I_A$ , and Cumulative Absolute Velocity  $CAV$ . These observations are meaningful as the objective of the proposed scaling procedure is to estimate EGM for nonlinear inelastic time-history analysis; hence, within the context of seismic structural design, a GMP well associated with ductility demand is what is desirable to guide amplitude-scaling.



**Figure 6.** Comparison of GMPs performance for a family of structures with periods between 0.1 and 2.0 sec and with a yield seismic coefficient  $C_y = 0.1$  (Martinez-Rueda and Vlachos, 2010).

Hence, the adopted definition for the amplitude-scaling factor is:

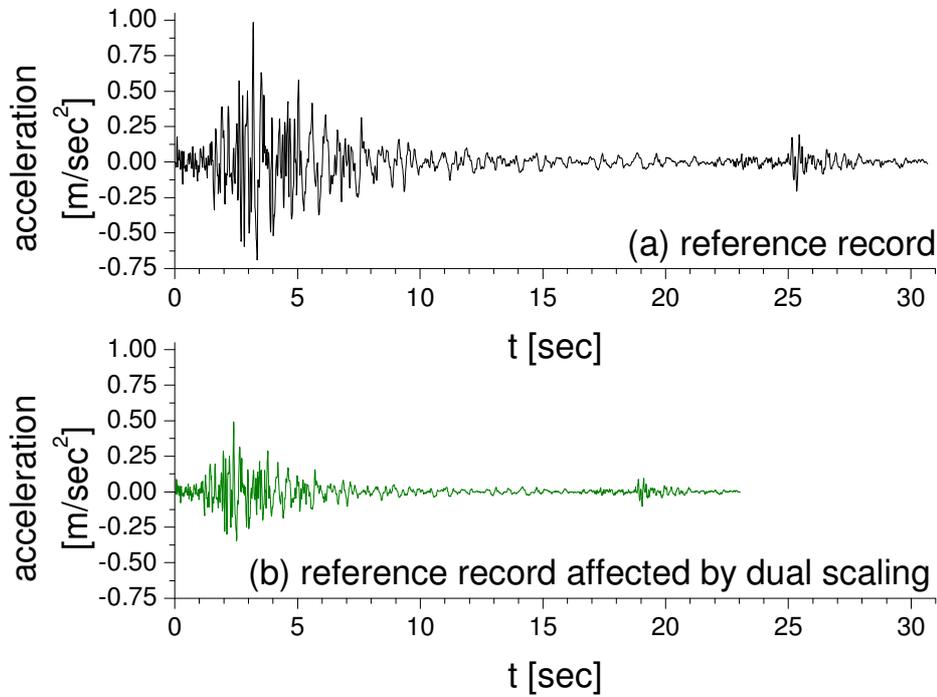
$$SF_a = \frac{SI_{Ht}}{SI'_{Href}} \quad (3)$$

where  $SI_{Ht}$  is the Housner Intensity of the predicted EGM used as target;  $SI'_{Href}$  is the Housner Intensity of the predicted EGM modified by time-scaling.

Based on the Housner intensities associated to the spectra of Figure 5,  $SF_a$  is finally calculated as:

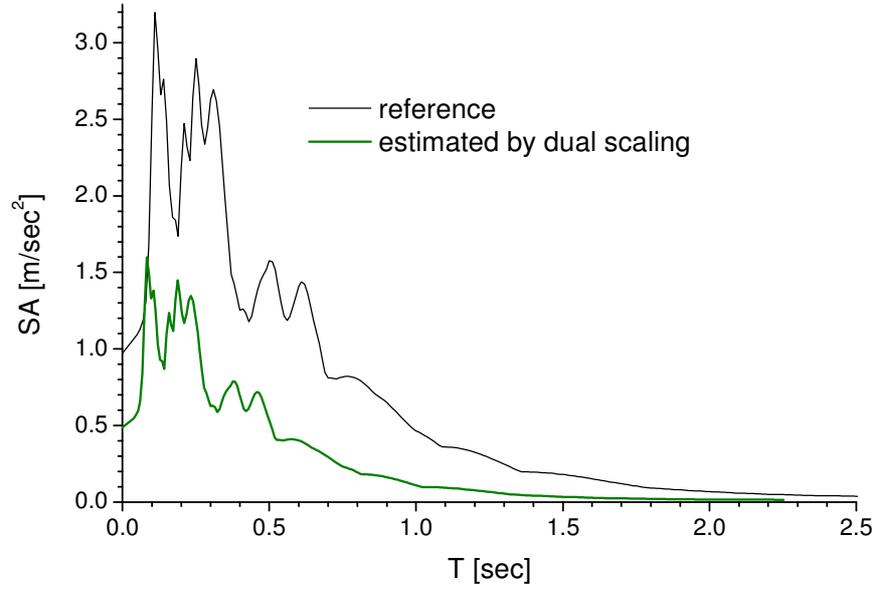
$$SF_a = \frac{0.0317 \text{ m/sec}}{0.0634 \text{ m/sec}} = 0.5$$

Multiplying by  $SF_t$  the time and by  $SF_a$  the acceleration of the REGM we obtain by dual scaling an estimated EGM for the target magnitude. Figure 7 compares the reference EGM and the estimated EGM for the target magnitude.



**Figure 7.** Comparison between reference and scaled acceleration records.

As expected, the estimated EGM is less intense and of shorter duration when compared to the REGM; however, both records exhibit the same style of natural randomness with respect to the progression of the excitation. It can be argued that, although this fact is frequently ignored in the debate, this is actually one of the advantages of scaling over other methods to generate synthetic accelerograms. With scaling, the natural style of randomness of an earthquake signal used as a ‘seed’ is preserved and there is no risk of producing unrealistic EGM histories that may lead to a very smooth response spectrum or a to a ground motion with an excessive number of intense peaks which lead to unrealistically high energy content.



**Figure 8.** Comparison between reference and scaled acceleration records. ( $M_w = 5.9$  for the reference record and  $M_w = 5$  for the estimated EGM by dual scaling)

Figure 8 confirms the effectiveness of the suggested dual-scaling process as it shows a similar trend of that observed in Figure 2(a), *i.e.* for the same  $d$ , seismic site and faulting style, a decrease in magnitude leads to a narrowing of the amplification band and to a decrease of the spectral ordinates. As expected, while both spectra display the same overall geometry, because of the time-scaling involved the positions of the spectrum peaks are not matched.

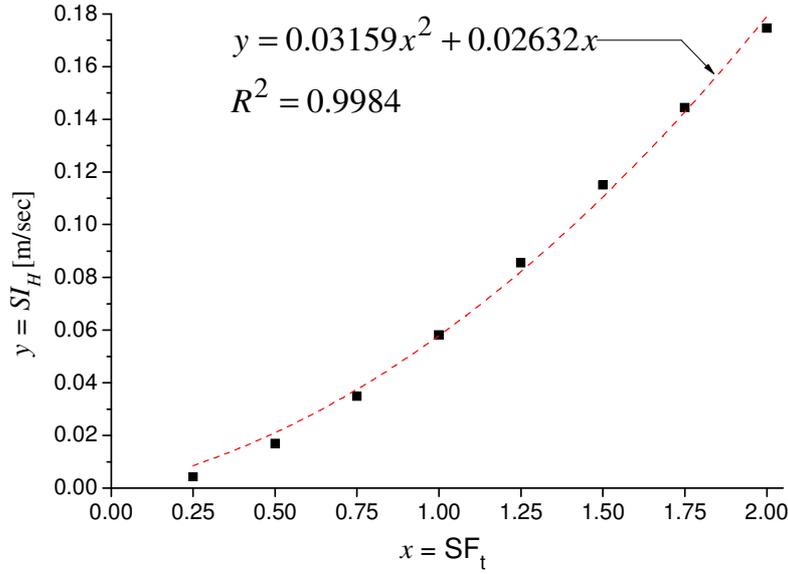
### 3.2 Dual scaling guided by GMPEs of seismic intensity assessed in period and time domains.

Traditionally, GMPEs had been implemented primarily for the estimation of  $PGA$  or  $SA$ . More recently however, a number of GMPEs to estimate other GMPs have begun to appear in the literature. For instance, a GMPE for Arias Intensity has already been proposed by Travarasrou, *et al.* (2003) and a semi-empirical GMPE for Housner Intensity was introduced by the author (Martinez-Rueda, 2006b). It is important to visualise that the above pair of GMPE were not calibrated using the same data set and region of the world so, in principle, as such they could not be used jointly for dual scaling. In the following discussion it is initially assumed that we count with a pair of GMPE to estimate Arias Intensity and Housner Intensity valid and calibrated both for the same region of the world.

Because of its fixed integration range, the effect of time-scaling on  $SI_H$  is not linear. In fact, as exemplified in Figure 9, the relationship between  $SF_t$  and  $SI_H$  may be approximated by a truncated polynomial of second degree which predicts a null intensity when  $SF_t = 0$  (*i.e.* for no ground motion); namely:

$$SI'_H = A \cdot SF_t^2 + B \cdot SF_t \quad (4)$$

where  $SI'_H$  is the Housner intensity affected by time scaling;  $A$  &  $B$  are fitting constants.



**Figure 9.**  $SF_t$  vs.  $SI_H$  relationship derived for the REGM of Figure 7(a).

To achieve a given target Housner intensity  $SI_{Ht}$  we may need to resort also to amplitude-scaling. If that is the case then eqn. (4) becomes:

$$SI_{Ht} = SF_a(A \cdot SF_t^2 + B \cdot SF_t) \quad (5)$$

If we also want to impose by dual scaling the achievement of a given target Arias intensity  $I_{At}$ , then the relationship between  $I_{At}$  and the Arias intensity of the reference record  $I_{Aref}$  can be expressed as:

$$I_{At} = SF_a^2 \cdot SF_t \cdot I_{Aref} \quad (6)$$

The solution of the nonlinear system of eqns (5)-(6) leads to the required scaling factors  $SF_t$  and  $SF_a$ .

As pointed out earlier, to assess the effectiveness of the system of eqns. (5)-(6) it could be very speculative to apply jointly a GMPE for  $I_A$  and another GMPE for  $SI_H$  if they have not been calibrated for the same dataset and accounting for an identical characterisation of the seismological parameters. Nevertheless, for illustrative purposes only, let us consider the same example of dual scaling covered in section 3.1. The estimated target Housner intensity predicted by the GMPE of Martinez-Rueda (2006) is  $SI_{Ht} = 0.0317 \text{ m/sec}^2$ . The estimated target and reference Arias intensities predicted by the GMPE of Travarasou, *et al.* (2003) are  $I_{At} = 0.0302 \text{ m/sec}^2$  and  $I_{Aref} = 0.1567 \text{ m/sec}^2$ , respectively. Using the above values as well as the empirical constants  $A$  and  $B$  calibrated in Figure 9 the solution of the system of eqns (5)-(6) gives the scaling factors:  $SF_a = 0.3960$  and  $SF_t = 1.2289$ . As expected, this solution is different from that obtained in section 3.1. The major difference occurs for the time-scaling factor. Furthermore, the dual scaling guided by eqns. (5)-(6) appears to contradict the physics of the problem under study as the estimated ground motion exhibits increased duration. One could argue that unlike the approach of section 3.1 (*i.e.* dual scaling guided by changes in SA spectrum), the approach for dual scaling of this section (*i.e.* dual scaling guided by changes in GMPs that assess intensity in the period and time domain) does not impose a control on the frequency content of the scaled record. This appears to be the reason behind the paradoxical value obtained for  $SF_t$ . In fact, Arias intensity explicitly accounts for amplitude and duration, but only implicitly accounts for frequency content and hence it is insensitive to the amplification band or to the predominant frequency/period of excitation. However, as pointed out earlier we need to be mindful that the GMPEs for Arias Intensity and Housner Intensity used in this example were not calibrated using the same dataset of natural accelerograms. Hence, there is not enough evidence to invalidate the applicability of the system of equations (5)-(6) to define  $SF_a$  and  $SF_t$ .

#### 4. CONCLUDING REMARKS

This study introduced two distinct approaches for the dual scaling of natural accelerograms to estimate the effects that changes of magnitude and distance to the source have on frequency content and intensity.

It is concluded that, at present, modern GMPEs for spectral acceleration offer an attractive option to guide the dual scaling of natural earthquake records. These GMPEs provide sufficient information for an explicit and unambiguous calibration of time-scaling and amplitude-scaling factors. In fact, this study showed that dual scaling can provide a good estimation of the overall changes in duration, frequency content and intensity of the seismic input produced by changes in magnitude and distance to the source for the same style of faulting and seismic site.

It is important to note that the dual scaling studied in this research made use of GMPEs which for their calibration records generated by aftershocks were excluded. Until further studies are conducted it is suggested that the reference earthquake record subjected to dual scaling has been recorded during a main event.

It is also suggested that a promising dual scaling procedure that may yield good results will be one which involves the use of a GMPE for a GMP that explicitly accounts for frequency content. The search of this alternative procedure as well as a detailed analysis of the implications of dual scaling on the estimation of ductility demands are topics currently under study by the author.

Finally, Bommer and Acevedo (2004) wisely warn about the possible imperfections that time-scaling may introduce to a natural acceleration record. In the author's experience however, if the reference record is of high quality (in terms of its base-line correction and filtering) time-scaling is not of major concern. Nevertheless, before using an acceleration record that has been produced by time-scaling, the engineer is expected to verify its adequacy by obtaining by integration the velocity and displacement time-histories of the ground motion. It is expected that in a few cases an additional polynomial base-line correction may be required to adjust the time-scaled record.

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